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New $D=4$ gauged supergravities from $\mathcal{N}=4$ orientifolds with fluxes

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Abstract: We consider classes of T_6-orientifolds, where the orientifold projection contains an inversion I_{9-p} on $9-p$ coordinates, transverse to a Dp-brane. In absence of fluxes, the massless sector of these models corresponds to diverse forms of $\mathcal{N}=4$ supergravity, with six bulk vector multiplets coupled to $\mathcal{N}=4$ Yang-Mills theory on the branes. They all differ in the choice of the duality symmetry corresponding to different embeddings of $\mathrm{SU}(1,1)\times\mathrm{SO}(6,6+n)$ in $\mathrm{Sp}(24+2n,\mathbb{R})$, the latter being the full group of duality rotations. Hence, these lagrangians are not related by local field redefinitions. When fluxes are turned on one can construct new gaugings of $\mathcal{N}=4$ supergravity, where the twelve bulk vectors gauge some nilpotent algebra which, in turn, depends on the choice of fluxes.

Keywords: D-branes, Supergravity Models.

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1. Introduction

New string or M-theory models are obtained turning on n-form fluxes, which allow, in general, the lifting of vacua, supersymmetry breaking and moduli stabilisation [1]–[24]. Examples of such new solutions are IIB and IIA orientifolds [25]–[29], where the orientifold projection (in absence of fluxes) preserves $\mathcal{N} = 4$ or $\mathcal{N} = 2$ supersymmetries.

Recently, the $T_6/\mathbb{Z}_2$ orientifold with $\mathcal{N} = 4$ supersymmetry [8, 9] and $K_3 \times T_2/\mathbb{Z}_2$ orientifold [24] with $\mathcal{N} = 2$ supersymmetry have been the subject of an extensive study. In these cases, turning on NS–NS and R–R three–form fluxes allows to obtain new string vacua with vanishing vacuum energy, reduced supersymmetry and moduli stabilisation [7]–[9] and [21, 24]. These features can all be understood in terms of an effective gauged supergravity, where certain axion symmetries are gauged [30]–[32]. These are generalised no-scale models [33, 34].

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In the present investigation, we consider more general four-dimensional orientifolds with fluxes (both in type IIB and IIA) where the orientifold projection involves an inversion I_{9-p} on $9-p$ coordinates, transverse to the Dp-brane world-volume, thus generalising the $T_6/\mathbb{Z}_2$ orientifold (with $p=3$) constructed by Frey-Polchinski [8] and Kachru-Shulz-Trivedi [9] (see also [10] for a derivation of the complete low-energy supergravity from T-dialysed Type I theory in ten dimensions). Interestingly, their low-energy descriptions are all given in terms of $\mathcal{N}=4$ supergravity with six vector supermultiplets from the closed-string sector, coupled to an $\mathcal{N}=4$ Yang-Mills theory living on the Dp-brane world-volume.

However, despite the uniqueness of $\mathcal{N}=4$ supersymmetry, the low-energy actions crucially differ in the choice of the manifest "duality symmetries" of the lagrangian, since different sets of fields survive the orientifold projection, and therefore different symmetries are manifestly preserved. Leaving the brane degrees of freedom aside, these duality symmetries are specified by their action on the (twelve) bulk vectors. Actually, $\mathcal{N}=4$ supergravity demands that such symmetries be contained in $\mathrm{SU}(1,1)\times\mathrm{SO}(6,6)$ [35, 36] and act on the vector field strengths and their duals as symplectic $\mathrm{Sp}(24,\mathbb{R})$ transformations [37]. On the other hand, the symmetries of the lagrangian correspond to block-lower-triangular symplectic matrices, whose block-diagonal components have a definite action on the vector potentials [38, 39, 40]. For instance, in the orientifold models containing an I_{9-p} inversion, the block-diagonal symmetries always include $\mathrm{GL}(9-p,\mathbb{R})\times\mathrm{GL}(p-3,\mathbb{R})$, as maximal symmetry of the $\mathrm{GL}(6,\mathbb{R})$ associated to the moduli space of the six-torus metrics. The lower-triangular block contains the axion symmetries of the R-R scalars and of the NS-NS ones originating from the B-field, whenever present[1].

In the sequel, we describe all nilpotent algebras N_p [41], corresponding to axion symmetries of the R-R and NS-NS scalars for all orientifold models. All N_p's are nilpotent subalgebras of so$(6,6)$, are generically non-abelian and contain central charges. There are four of them in type IIB ($p=3$, 5, 7, 9) with dimensions 15, 23, 23, 15 respectively, while there are only three of them in type IIA ($p=4$, 6, 8) of dimensions 20, 24, 20, respectively. A common feature of these algebras is that they always contain fifteen R-R axionic symmetries, while the extra symmetries correspond to NS-NS B-field axions in the bi-fundamental of $\mathrm{GL}(9-p,\mathbb{R})\times\mathrm{GL}(p-3,\mathbb{R})$.

A further R-R axion symmetry originates from the $\mathrm{SU}(1,1)$, which acts as electric-magnetic duality on the gauge fields living on the brane world-volume. The corresponding axion field can be identified with the C_{p-3} R-R field, as dictated by the coupling

$$\int_{\Sigma_{p+1}} C_{p-3}\wedge F\wedge F\,, \tag{1.1}$$

where F is the two-form field strength of gauge fields living on the branes.

Turning on fluxes in the orientifold models (three- and five-form fluxes in type IIB, two- and four-form fluxes in IIA) corresponds to a "gauging" in the corresponding supergravity lagrangian, whose couplings are dictated by the particular choice of fluxes. Non-abelian gaugings may also occur corresponding to subalgebras of N_p, or quotient algebras N_p/Z, where Z are some of the central generators of N_p.

[1] For example, the latter is not present in the $p=3$ case, i.e. the $T_6/\mathbb{Z}_2$ orientifold.

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As an illustrative example, let us consider the $p = 7$ type IIB orientifold defined in section 2, where the non-vanishing NS-NS and R-R fluxes are H_{aij}, F_{aij}, $G^i = \epsilon^{ab}\,\epsilon^{ijkl}\,G_{abjkl}$ ($a, b = 5, 6$ and $i, j = 1, \ldots, 4$), and let us look at terms involving the axions coming from the B and four-form fields, B_{ia} and $C_{ijab} = C_{ij}\epsilon_{ab}$. Inspection of the three-form kinetic term reveals a non-abelian gauge coupling proportional to

$$\sqrt{-g}\, H_{aij}\, H_{\mu\nu b}\, g^{ab}\, g^{i\mu}\, g^{j\nu}\,, \tag{1.2}$$

as well as axion gauge couplings proportional to

$$\sqrt{-g}\, H_{aij}\, H_{\mu b\ell}\, g^{ab}\, g^{i\mu}\, g^{j\ell}\,, \tag{1.3}$$

together with similar expressions for the F-three form. Such terms come also from the reduction of type IIB four-form field. In addition, when a five-form flux G^i is turned on an axion gauge coupling emerges of the type

$$\partial_\mu C_{ij} + \epsilon_{ijk\ell}\, G^k\, G^\ell_\mu\,. \tag{1.4}$$

where $G^\ell_\mu = g^{\ell i}\, g_{i\mu}$ are the Kaluza-Klein vectors. We report here only a preliminary analysis of the deformation of the $\mathscr{N} = 4$ supergravity due to these new gaugings.

In the present paper we do not address either the question of unbroken supersymmetries or the question of moduli stabilisation, which would require the knowledge of the scalar potential and a study of the fermionic sector. However, we can anticipate that certain moduli are indeed stabilised in all these models, since a Higgs effect is taking place as suggested by the presence of charged axion couplings.

The paper is organised as follows: in section 2, we review the four-dimensional $T_6/\mathbb{Z}_2$ orientifold models, their spectra and their allowed fluxes. In section 3, the $\mathscr{N} = 4$ supergravity interpretation is given for the ungauged case (absence of fluxes) and the duality symmetries exposed. The N_p algebras are exhibited as well as their action on the vector fields. In section 4, we give a preliminary description of gauged supergravity, for the particular case of type IIB orientifolds with some three-form fluxes turned on. In section 5 some conclusions are drawn. Finally, in appendix some useful formulae needed to compute the quadratic part of the vector field strengths in the lagrangian, are given.

2. $\mathscr{N} = 4$ orientifolds: spectra and fluxes

In this section we review the construction of orientifold models preserving $\mathscr{N} = 4$ supersymmetries in $D = 4$ [29]. This is the simplest setting for orientifold constructions, and consists of modding out type II superstrings by the world-sheet parity Ω [25]. Following [28, 29], the orientifold projection can be given a suggestive geometrical interpretation in terms of non-dynamical defects, the orientifold $\mathscr{O}$-planes, that reflect the left-handed and right-handed modes of the closed string. Actually, one can combine world-sheet parity with other (geometrical) operations. In general, this can affect the nature of the orientifold planes, that, in the simplest instance of a bare Ω have negative tension and R-R charge, and are $(9 + 1)$-dimensional ($\mathscr{O}9$ planes) since they have to respect the full Lorentz symmetry

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preserved by Ω. In the present paper, we are interested in the class of models generated by the ΩI_{9-p} generator, where I_{9-p} denotes the inversion on $9-p$ coordinates. Of course, ΩI_{9-p} must be a symmetry of the parent theory, and this is the case of type IIB for p odd, and of type IIA for p even. Actually, ΩI_{9-p} reflects the action of T-duality in orientifold models. Indeed, T-duality itself can be thought of as a chiral parity transformation

$$X_{\rm L} \to X_{\rm L}\,, \qquad X_{\rm R} \to -X_{\rm R}\,, \tag{2.1}$$

and conjugates Ω so to get

$$\mathscr{T}_{9-p}\Omega\mathscr{T}_{9-p}^{-1} = \Omega I_{9-p}\,. \tag{2.2}$$

As a result, the full ten-dimensional Lorentz symmetry is now broken to the subgroup $\mathrm{SO}(1,p)\times\mathrm{SO}(9-p)$, and the closed-string sector involves $\mathscr{O}_{9-p}$ planes sitting at the fixed points of the orbifold T_{9-p}/I_{9-p}. The associated open-string sector will then correspond to open strings with Dirichlet boundary conditions along T_{9-p}, i.e. open strings ending on $\mathrm{D}(9-p)$ branes. As usual, tadpole conditions will fix the rank of the Chan-Paton gauge group, i.e. the total number of D-branes. In the present paper, however, we shall not be concerned with open-string degrees of freedom and we shall concentrate our analysis solely on the closed-string degrees of freedom.

Before we turn to the description of specific models, a general comment is in order. An important requirement in the construction is that the orientifold group be $\mathbb{Z}_2$, i.e. its generator ΩI_{9-p} must square to the identity. Although Ω has always ±1 eigenvalues, and thus $\Omega^2=1$, this is not the case for I_{9-p}. For example, for $p=7$ I_2 would correspond to a π rotation on a two-plane and, although its action on the bosonic degrees of freedom is real and assigns to them a plus or minus sign according to the number of indices along the two-plane, its eigenvalue on spinors is $e^{i\pi\Sigma}$, where $\Sigma=\pm\frac{1}{2}$ are the two helicities. Thus, it does not square to the identity, but rather to $(-1)^F$, with F the (total) space-time fermion number. Therefore, in this case the orientifold projection needs be modified by the inclusion of $(-1)^{F_{\rm L}}$, with $F_{\rm L}$ the left-handed space-time fermion number [47]. We are thus dealing with the four-dimensional orientifolds

$$\left(T_{p-3}\times T_{9-p}\right)/\,\Omega I_{9-p}\left[(-1)^{F_{\rm L}}\right]^{\left[\frac{9-p}{2}\right]}\,, \tag{2.3}$$

where $[\frac{9-p}{2}]$ denotes the integer part of $(9-p)/2$. Here we have decomposed the six-torus as

$$T_6 = T_{p-3}\times T_{9-p}\,, \tag{2.4}$$

since I_{9-p} only acts on the coordinates of T_{9-p}, while leaves invariant those along T_{p-3}. As we shall see, this is a natural decomposition since, in the orientifold, we are left with the perturbative symmetry $\mathrm{GL}(p-3)\times\mathrm{GL}(9-p)$ of the compactification torus. To fix the notation, in this paper we shall label coordinates on the T_6 with a pair of indices (i,a), where $i=1,\ldots,p-3$ counts the coordinates not affected by the space parity (those coordinates that would be longitudinal to the branes), while $a=1,\ldots,9-p$ runs over the coordinates of T_{9-p} (orthogonal to the branes). As usual, Greek indices $\mu,\nu,\ldots$ will label coordinates on the four-dimensional Minkowski space-time.

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p	scalars	vectors
9	g_{ij}, ϕ, $C_{\mu\nu}$, C_{ij}	G^i_μ, $C_{i\mu}$
7	g_{ij}, g_{ab}, ϕ, B_{ia}, C, C_{ia}, C_{ijkl}, C_{ijab}	G^i_μ, $B_{a\mu}$, $C_{a\mu}$, $C_{ijk\mu}$
5	g_{ij}, g_{ab}, ϕ, B_{ia}, $C_{\mu\nu}$, C_{ij}, C_{ab}, C_{iabc}	G^i_μ, $B_{a\mu}$, $C_{i\mu}$, $C_{abc\mu}$
3	g_{ab}, ϕ, C, C_{abcd}	$B_{a\mu}$, $C_{a\mu}$

Table 1: Massless degrees of freedom for the IIB orientifolds.

At this point, it is better to consider the cases p odd or p even separately. In the first case, $\Omega I_{9-p}[(-1)^{F_{\rm L}}]^{[\frac{9-p}{2}]}$ is a symmetry in type IIB, while in the latter case it is properly defined within type IIA.

2.1 IIB orientifolds

In type IIB superstring we have to consider four cases, corresponding to the allowed choices $p = 9, 7, 5, 3$. The massless ten-dimensional fields have a well defined parity with respect to Ω:

$$\text{even}: \quad G_{MN}\,, \quad \phi\,, \quad C_{MN}\,, \tag{2.5}$$

$$\text{odd}: \quad B_{MN}\,, \quad C\,, \quad C^{(+)}_{MNPQ}\,, \tag{2.6}$$

where G_{MN} is the metric tensor, ϕ the dilaton, B_{MN} the Kalb-Ramond two-form, and C_{p+1} are the R-R $(p+1)$-forms.[2] Henceforth, it is straightforward to select the four-dimensional excitations that survive the orientifold projection. In fact, after splitting the ten-dimensional index M in the triple (μ, i, a) labelling $\mathscr{M}_{1,3} \times T_{p-3} \times T_{9-p}$, it is evident that the fields with an odd (even) number of a-type indices are odd (even) under the action of I_{9-p}. On the other hand, when present, $(-1)^{F_{\rm L}}$ assigns a plus sign to the NS-NS states (which originate from the decomposition of the product of two bosonic representations of SO(8)) and a minus sign to the R-R states (which originate from the decomposition of the product of two spinorial representations of SO(8)). At the end, aside from the four-dimensional metric tensor, one is left with the massless (bosonic) degrees of freedom listed in table 1.

p	fluxes
9	none
7	H_{ija}, F_{ija}, G_{ijkab}
5	H_{abc}, F_{iab}, H_{ija}, G_{ijabc}
3	H_{abc}, F_{abc}

Table 2: Allowed fluxes for the IIB orientifolds. F, H and G fluxes are associated to the B, C_2 and C_4 fields.

However, in orientifold models it happens often that fields which are odd under the projection can be consistently assigned with a (quantised) background value for the fields themselves, or for their field strengths. For example, in the $p = 7$ case the NS-NS fields B_{ij} and the R-R fields C_{ij} are both odd with respect to the orientifold projection and, thus, their quantum excitations are projected out. However, acting on them with a ∂_a derivative changes their parity, and thus (quantised) fluxes along the internal directions, H_{aij} and F_{aij}, can be incorporated in the model. Repeating a similar analysis for the other cases yields the allowed fluxes listed in table 2.

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p	scalars	vectors
8	g_{ij}, g_{99}, ϕ, B_{i9}, C_i, $C_{9\mu\nu}$, C_{ij9}	G^i_μ, C_μ, $C_{i9\mu}$, $B_{9\mu}$
6	g_{ij}, g_{ab}, ϕ, B_{ia}, C_a, $C_{i\mu\nu}$, C_{ijk}, C_{iab}	G^i_μ, $B_{a\mu}$, $C_{ij\mu}$, $C_{ab\mu}$
4	g_{44}, g_{ab}, ϕ, B_{4a}, C_4, $C_{a\mu\nu}$, C_{abc}	G^4_μ, $B_{a\mu}$, C_μ, $C_{4a\mu}$

Table 3: Massless degrees of freedom for the IIA orientifolds.

2.2 IIA orientifolds

Type IIA superstring selects p even, and thus leaves us with the three cases $p = 8, 6, 4$. Although a bare Ω is not a symmetry in type IIA, we can nevertheless assign a well defined parity to the massless ten-dimensional degrees of freedom:

p	fluxes
8	H_{ij9}, G_{ijk9}
6	H_{aij}, H_{abc}, F_{ia}, G_{ijab}
4	H_{abc}, F_{ab}, G_{4abc}

Table 4: Allowed fluxes for the IIA orientifolds. F, H and G fluxes are associated to the B, C_1 and C_3 fields.

$$\text{even}: \quad G_{MN}\,, \quad \phi\,, \quad C_M\,, \tag{2.7}$$

$$\text{odd}: \quad B_{MN}\,, \quad C_{MNP}\,. \tag{2.8}$$

As before, G_{MN} is the metric tensor, ϕ the dilaton, B_{MN} the Kalb-Ramond two-form, while in this case the R-R potentials C_{p+1} carry an odd number of indices. The additional action of I_{9-p} and, eventually, of $(-1)^{F_{\rm L}}$ thus yields the massless degrees of freedom listed in table 3.

Also in this case one can allow for (quantised) fluxes along the compactification torus, as summarised in table 4.

3. $\mathcal{N} = 4$ supergravity interpretation of T_6 orientifolds: manifest duality transformations and Peccei-Quinn symmetries

The four-dimensional low-energy supergravities of $\mathcal{N} = 4$ orientifolds (in the absence of fluxes) can be consistently constructed as truncations of the unique four-dimensional $\mathcal{N} = 8$ supergravity which describes the low-energy limit of dimensionally reduced type II superstrings. Its duality symmetry group $E_{7(7)}$ acts non linearly on the 70 scalar fields, and linearly, as a $\mathrm{Sp}(56, \mathbb{R})$ symplectic transformation, on the 28 electric field strengths and their magnetic dual. In this framework an intrinsic group-theoretical characterisation of the ten-dimensional origin of the four-dimensional fields is indeed achieved. In the so-called solvable Lie algebra representation of the scalar sector [41, 42], the scalar manifold

$$\mathcal{M}_{\rm scal} = \exp\left(\mathrm{Solv}(e_{7(7)})\right) \tag{3.1}$$

is expressed as the group manifold generated by the solvable Lie algebra $\mathrm{Solv}(e_{7(7)})$ defined through the Iwasawa decomposition of the $e_{7(7)}$ algebra:

$$e_{7(7)} = su(8) + \mathrm{Solv}\left(e_{7(7)}\right). \tag{3.2}$$

In this framework, there is a natural one-to-one correspondence between the scalar fields and the generators of $\mathrm{Solv}(e_{7(7)})$. The latter consists of the 7 generators H_p of the $e_{7(7)}$ Cartan subalgebra, parametrised by the T_6 radii $R_n = e^{\sigma_n}$ together with the dilaton ϕ,

[2]Actually, the four-form $C_4^{(+)}$ is constrained to have a self-dual field strength, a peculiarity of type IIB

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and of the shift generators corresponding to the 63 positive roots α of $e_{7(7)}$, which are in one-to-one correspondence with the axionic scalars that parametrise them. This correspondence between Cartan generators and positive roots on one side and scalar fields on the other, can be pinpointed by decomposing $\mathrm{Solv}(e_{7(7)})$ with respect to some relevant groups. For instance, the duality group of maximal supergravity in D dimensions is $E_{11-D(11-D)}$ and therefore, in the solvable Lie algebra formalism, the scalar fields in the D-dimensional theory are parameters of $\mathrm{Solv}(e_{11-D(11-D)})$. Since $e_{11-D(11-D)} \subset e_{7(7)}$, decomposing $\mathrm{Solv}(e_{7(7)})$ with respect to $\mathrm{Solv}(e_{11-D(11-D)})$ it is possible to characterise the higher-dimensional origin of the four-dimensional scalars. Moreover, in four dimensions the group $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SO}(6,6)_T \subset E_{7(7)}$, $\mathrm{SO}(6,6)_T$ being the isometry group of the T_6 moduli-space, acts transitively on the scalars originating from ten-dimensional NS-NS fields of type II theories. These scalars therefore parametrise $\mathrm{Solv}(\mathrm{sl}(2,\mathbb{R}) + \mathrm{so}(6,6)_T)$. Henceforth, decomposing $\mathrm{Solv}(e_{7(7)})$ with respect to $\mathrm{Solv}(\mathrm{sl}(2,\mathbb{R}) + \mathrm{so}(6,6)_T)$ one can achieve an intrinsic characterisation of the NS-NS or R-R ten-dimensional origin of the four-dimensional scalar fields, the R-R scalars (and the corresponding solvable generators) transforming in the spinorial representation of $\mathrm{SO}(6,6)_T$. Finally, depending on whether we interpret the four-dimensional maximal supergravity as tied to type II supergravities on T_6 or $D = 11$ supergravity on T_7, the metric moduli are acted on transitively by $\mathrm{GL}(6,\mathbb{R})_g$ or $\mathrm{GL}(7,\mathbb{R})_g$ subgroups of $E_{7(7)}$, respectively. Therefore, in the two cases the metric moduli parametrise $\mathrm{Solv}(gl(6,\mathbb{R})_g)$ or $\mathrm{Solv}(gl(7,\mathbb{R})_g)$ and thus, decomposing $\mathrm{Solv}(e_{7(7)})$ with respect to these two solvable subalgebras, depending on the higher-dimensional interpretation of the four-dimensional theory, we may split the axions into metric moduli of the internal torus and into scalars deriving from dimensional reductions of ten- or eleven-dimensional tensor fields. The latter will parametrise nilpotent generators transforming in the corresponding tensor representations with respect to the adjoint action of $\mathrm{GL}(6,\mathbb{R})_g$ or $\mathrm{GL}(7,\mathbb{R})_g$. As a result of the above decompositions, we are able to characterise unambiguously each parameter of $\mathrm{Solv}(e_{7(7)})$ as a dimensionally reduced field. Let us consider the dimensional reduction of type II supergravities. As far as the axionic scalars are concerned the correspondence with roots can be summarised in terms of an orthonormal basis $\{\epsilon_p\}$ of $\mathbb{R}^7$:[3]

$$C_{n_1 n_2 \dots n_k} \leftrightarrow a + \epsilon_{n_1} + \cdots \epsilon_{n_k} \,, \tag{3.3}$$

$$C_{n_1 n_2 \dots n_k \mu\nu} \leftrightarrow a + \epsilon_{m_1} + \cdots \epsilon_{m_{6-k}} \,, \qquad (\epsilon^{n_1 \dots n_k m_1 \dots m_{6-k}} \neq 0) \,, \tag{3.4}$$

$$B_{nm} \leftrightarrow \epsilon_n + \epsilon_m \,, \tag{3.5}$$

$$B_{\mu\nu} \leftrightarrow \sqrt{2}\,\epsilon_7 \,, \tag{3.6}$$

$$G_{nm} \leftrightarrow \epsilon_n - \epsilon_m \,, \qquad (n \neq m) \,, \tag{3.7}$$

where

$$a = -\frac{1}{2}\sum_{n=1}^{6} \epsilon_n + \frac{1}{\sqrt{2}}\epsilon_7 \,. \tag{3.8}$$

[3] Now and henceforth we shall always label by n, $m = 1, \dots, 6$ the T_6 directions, by i, $j = 1, \dots, p-3$ the directions of T_{p-3} which are longitudinal to the Dp-brane and by a, $b = p-2, \dots, 9-p$ the directions of the transverse T_{9-p}. The four-dimensional space-time directions are generically denoted by Greek letters.

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In our notation, the $\mathrm{so}(6,6)_T$ roots have the form $\{\pm\epsilon_n \pm\epsilon_m\}$, where $1 \le n < m \le 6$. Notice indeed that the nilpotent generators corresponding to non-metric axions transform in tensor representations of $\mathrm{GL}(6,\mathbb{R})_g$, and this, in turn, defines the $\mathrm{GL}(6,\mathbb{R})_g$ representation of the corresponding scalar. For instance, the $C_{n_1\dots n_k}$ parametrises the generator $T^{n_1\dots n_k} = E_{a+\epsilon_{n_1}+\dots\epsilon_{n_k}}$ whose transformation property under $\mathrm{GL}(6,\mathbb{R})_g$ is

$$\mathbf{g} \in \mathrm{GL}(6,\mathbb{R})_g : \quad \mathbf{g}\cdot T^{n_1\dots n_k}\cdot \mathbf{g}^{-1} = g^{n_1}{}_{m_1}\cdots g^{n_k}{}_{m_k}\, T^{m_1\dots m_k} . \tag{3.9}$$

The roots corresponding to R-R fields are spinorial with respect to $\mathrm{SO}(6,6)_T$ and, depending on whether the number of their indices is even or odd, they belong to the root system of two $e_{7(7)}$ algebras which are mapped into each other by the $\mathrm{SO}(6,6)_T$ outer automorphism (T-duality) [43, 44]. These two systems naturally correspond to the reduction of IIB and IIA superstrings, that are indeed related by T-dualities. Hence, the T_6 metric moduli in the type IIA or B descriptions, are acted upon transitively by two inequivalent $\mathrm{GL}(6,\mathbb{R})_g$ subgroups of $E_{7(7)}$: in the former case $\mathrm{GL}(6,\mathbb{R})_g$ is contained in $\mathrm{SL}(8,\mathbb{R}) \subset E_{7(7)}$, while in the latter case $\mathrm{GL}(6,\mathbb{R})_g$ is contained in the maximal subgroup $\mathrm{SL}(3,\mathbb{R}) \times \mathrm{SL}(6,\mathbb{R})_g$ of $E_{7(7)}$. As far as the R-R scalars are concerned, the two representations differ in the $\mathrm{SO}(6,6)_T$ chirality of the 32 spinorial positive roots

$$\begin{aligned}
\mathrm{IIA} : \mathbf{32}^- &= \left\{ \frac{1}{2}(\overbrace{\pm\epsilon_1 \dots \pm\epsilon_6}^{\text{odd }+}) + \frac{1}{\sqrt{2}}\,\epsilon_7 \right\} , \\
\mathrm{IIB} : \mathbf{32}^+ &= \left\{ \frac{1}{2}(\overbrace{\pm\epsilon_1 \dots \pm\epsilon_6}^{\text{even }+}) + \frac{1}{\sqrt{2}}\,\epsilon_7 \right\} .
\end{aligned} \tag{3.10}$$

Similarly, vector potentials, and their corresponding duals, are in one-to-one correspondence with weights W of the $\mathbf{56}$ of $E_{7(7)}$ in the two representations discussed above:

$$\begin{aligned}
C_{n_1\dots n_k\mu} &\leftrightarrow w + \epsilon_{n_1} + \cdots \epsilon_{n_k} , \\
B_{m\nu} &\leftrightarrow \epsilon_n - \frac{1}{\sqrt{2}}\epsilon_7 , \\
G^n_\mu &\leftrightarrow -\epsilon_n - \frac{1}{\sqrt{2}}\epsilon_7 ,
\end{aligned}$$

where

$$w = -\frac{1}{2}\sum_{n=1}^{6} \epsilon_n . \tag{3.11}$$

The dual potentials correspond to the opposite weights $-W$.

The above axion-root ($\Phi \leftrightarrow \alpha$) and vector-weight ($A_\mu \leftrightarrow W$) correspondences can be retrieved also from inspection of the scalar and vector kinetic terms in the dimensionally reduced type IIA or type IIB lagrangians [43, 45, 46] on a straight torus, which have the

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form:

$$\begin{aligned} \text{dilatonic scalars:} &\quad -\partial_\mu \vec{h} \cdot \partial^\mu \vec{h}\,, \\ \text{axionic scalars:} &\quad -\frac{1}{2}\, e^{-2\,\alpha\cdot h}\, \left(\partial_\mu \Phi \cdot \partial^\mu \Phi\right)\,, \\ \text{vector fields:} &\quad -\frac{1}{4}\, e^{-2\,W\cdot h}\, F_{\mu\nu}\, F^{\mu\nu}\,, \end{aligned}$$

where

$$\vec{h} = \sum_{n=1}^{6} \sigma_n \left(\epsilon_n + \frac{1}{\sqrt{2}}\epsilon_7\right) - \frac{1}{2}\,\phi\, a\,, \tag{3.12}$$

and, as usual, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

A generic axion Φ and its dilatonic partner $e^{\alpha\cdot h}$ can be thought of as the real and imaginary parts of a complex field z spanning an $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{SO}(2)$ submanifold, where the $\mathrm{SL}(2,\mathbb{R})$ group is defined by the root α. In the models describing type II strings on $T_{p-3} \times T_{9-p}$ orientifolds, the real part of the complex scalar z spanning the $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{SO}(2)$ factor in the scalar manifold is $C_{i_1\dots i_{p-3}}$, where $i_1,\dots i_k$ label the directions of T_{p-3}, as dictated by the coupling in eq. (1.1). From eqs. (3.3) and (3.12) one can then verify that $\mathrm{Im}(z) = e^{\alpha\cdot h} = \mathrm{Vol}_{p-3}\, e^{\frac{p-7}{4}\,\phi}$, where Vol_{p-3} denotes the volume of T_{p-3}. The scalar $\mathrm{Im}(z)$ defines the effective four-dimensional coupling constant of the super Yang-Mills theory on Dp-branes through the relation:

$$\frac{1}{g^2_{\mathrm{YM}}} = V_{p-3}\, e^{\frac{p-7}{4}\,\phi}\,. \tag{3.13}$$

The embedding of the $\mathcal{N} = 4$ orientifold models $T_{p-3} \times T_{9-p}$ (in absence of fluxes) inside the $\mathcal{N} = 8$ theory (in its type IIA or IIB versions) is defined by specifying the embedding of the $\mathcal{N} = 4$ duality group $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SO}(6,6)$ inside the $\mathcal{N} = 8$ $E_{7(7)}$ one. As far as the scalar sector is concerned, this embedding is fixed by the following group requirement:

$$\mathrm{SO}(6,6) \,\cap\, \mathrm{GL}(6,\mathbb{R})_g = \mathrm{O}(1,1) \times \mathrm{SL}(p-3,\mathbb{R}) \times \mathrm{SL}(9-p,\mathbb{R})\,. \tag{3.14}$$

Condition (3.14) fixes the ten-dimensional interpretation of the fields in the ungauged $\mathcal{N} = 4$ models (except for the cases $p = 3$ and $p = 9$) which, for a given p, is indeed consistent with the bosonic spectrum resulting from the orientifold reductions listed in the previous section. In the $p = 3$ and $p = 9$ cases, the two embeddings are characterised by a different interpretation of the scalar fields, consistent with the $T_6/\mathbb{Z}_2$ orientifold reduction in the presence of $D3$ or $D9$ branes. We shall denote these two models by $T_0 \times T_6$ and $T_6 \times T_0$, respectively. In these cases, equation (3.14) in the solvable Lie algebra language amounts to requiring that metric moduli are related either to the T_{p-3} metric g_{ij} or to the T_{9-p} metric g_{ab}. The scalar field parameterising the Cartan generator of the external $\mathrm{SL}(2,\mathbb{R})$ factor is given in eq. (3.13), while the metric modulus corresponding to the $\mathrm{O}(1,1)$ in eq. (3.14) is (modulo an overall power)

$$\mathrm{O}(1,1) \leftrightarrow (V_{p-3})^{9-p}\,(V_{9-p})^{11-p}\,. \tag{3.15}$$

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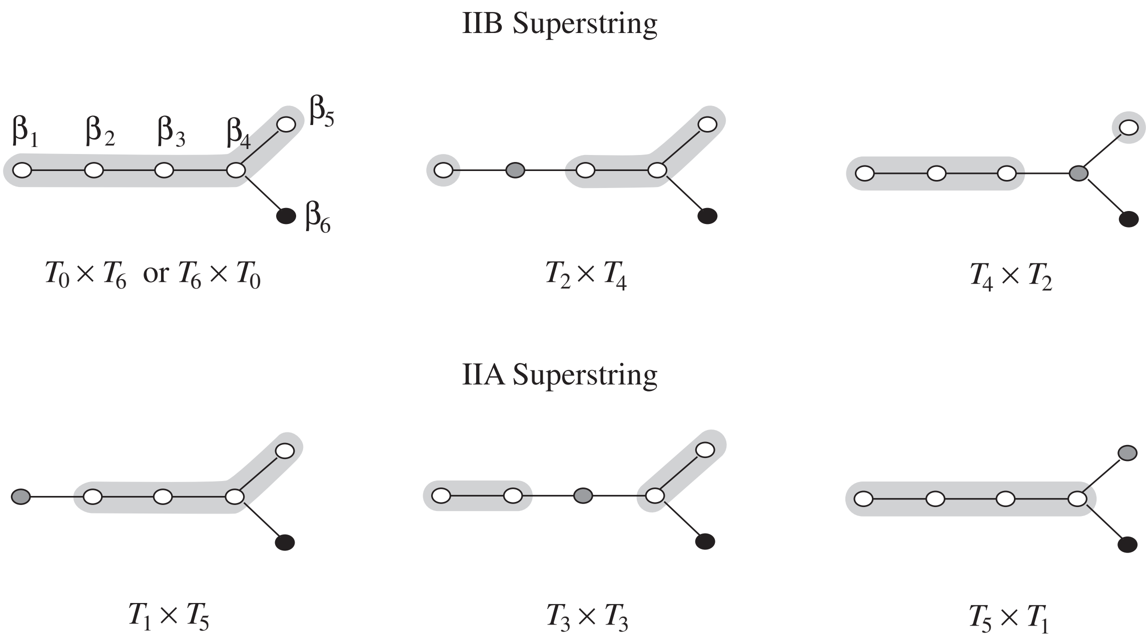

Figure 1: SO(6, 6) Dynkin diagrams for the $T_{p-3} \times T_{9-p}$ models. The shaded subdiagrams define the groups $\mathrm{SL}(p-3,\mathbb{R}) \times \mathrm{SL}(9-p,\mathbb{R})$ acting transitively on the metric moduli. The empty circles define simple roots corresponding to the metric moduli g_{ij}, g_{ab}, the grey circle denotes a simple root corresponding to a Kalb-Ramond field B_{ia} and the black circle corresponds to a R-R axion.

The axions not related to the T_6 metric moduli consist of $C_{i_1\dots i_{p-3}}$ in the external $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ factor, $(p-3)\,(9-p)$ moduli B_{ia} in the bifundamental of $\mathrm{SL}(p-3,\mathbb{R}) \times \mathrm{SL}(9-p,\mathbb{R})$ and 15 R-R moduli which we shall generically denote by C_I and which span the maximal abelian ideal $\{T^I\}$ of Solv(so(6, 6)). The scalars B_{ia} and C_I parametrise a $15+(p-3)\,(9-p)$ dimensional subalgebra N_p of Solv(so(6, 6)) consisting of nilpotent generators only. In figure 1 the so(6, 6) Dynkin diagrams for the various models and the corresponding intersections with $gl(6,\mathbb{R})_g$, represented by $\mathrm{sl}(p-3,\mathbb{R}) + \mathrm{sl}(9-p,\mathbb{R})$ subdiagrams, are illustrated. As far as the scalar fields are concerned, the $T_{p-3} \times T_{9-p}$ models within the same type IIA or IIB framework are mapped into each other by $\mathrm{so}(6,6)_T$ Weyl transformations, which can be interpreted as T-dualities on an even number of directions of T_6.

We con now turn to the detailed analysis of each (IIB or IIA) orientifold model.

3.1 $T_4 \times T_2$ IIB orientifold with D7 branes. The ungauged version

Solvable algebra of global symmetries. The following model (with $p = 7$) describes the bulk sector of IIB superstring compactified on a $(T_4 \times T_2)/\mathbb{Z}_2$ orientifold with D7 branes wrapped on the T_4.

To this end, we describe the embedding of the scalar sector of the corresponding $\mathscr{N} = 4$ model within the $\mathscr{N} = 8$ by expressing the so(6, 6) Dynkin diagram $\{\beta_n\}$ in terms of the

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simple roots of $e_{7(7)}$[4]

$$\begin{aligned}
\beta_1 &= \epsilon_1 - \epsilon_2\,, \\
\beta_2 &= \epsilon_2 - \epsilon_3\,, \\
\beta_3 &= \epsilon_3 - \epsilon_4\,, \\
\beta_4 &= \epsilon_4 + \epsilon_5\,, \\
\beta_5 &= -\epsilon_5 + \epsilon_6\,, \\
\beta_6 &= -\frac{1}{2}\left(\sum_{n=1}^{6}\epsilon_n\right) + \frac{1}{\sqrt{2}}\epsilon_7 = a\,.
\end{aligned}$$

According to eq. (3.3), the root β_6 corresponds to the ten-dimensional R-R scalar C_0, and thus identifies the type IIB duality group $\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$. The Dynkin diagram of the external $\mathrm{SL}(2,\mathbb{R})$ factor in the isometry group consists, instead, of the single root

$$\beta = a + \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4\,. \tag{3.16}$$

It is useful to classify the positive roots according to their grading with respect to three relevant $\mathrm{O}(1,1)$ groups generated by the Cartan operators H_β, H_{λ^4}, H_{λ^6} and parametrised by the moduli $\beta\cdot h$, h_4, h_6:

$$\begin{aligned}
\mathrm{O}(1,1)_0 &\to e^{\beta\cdot h} = V_4\,, \\
\mathrm{O}(1,1)_1 &\to e^{h_4} = (V_4)^{1/4}\,(V_2)^{1/2}\,, \\
\mathrm{O}(1,1)_2 &\to e^{h_6} = e^{-\phi}\,,
\end{aligned} \tag{3.17}$$

where we have denoted by λ^n the so(6,6) simple weights, $\lambda^n\cdot\beta_m = \delta^n_m$. $\mathrm{O}(1,1)_0$ is generated by the Cartan generator of the external $\mathrm{SL}(2,\mathbb{R})$ and $\mathrm{O}(1,1)_1$, $\mathrm{O}(1,1)_2$ are in $\mathrm{GL}(4,\mathbb{R})\times\mathrm{GL}(2,\mathbb{R})$, the former corresponding to the metric modulus given in eq. (3.15). In table 5 we list the axionic fields of the model together with the corresponding generator of $\mathrm{Solv}(\mathrm{sl}(2,\mathbb{R})) + \mathrm{Solv}(\mathrm{so}(6,6))$, for each of which the $\mathrm{O}(1,1)^3$ grading and the $\mathrm{SL}(4,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$ representations are specified. The indices i, j and a, b label as usual the directions of the torus which are longitudinal (T_4) and transverse (T_2) to the D-branes.

The fields B_{ia} and C_{ia} transform in the representation $(\mathbf{4},\mathbf{4})$ of $\mathrm{SL}(4,\mathbb{R})\times\mathrm{SO}(2,2)$ where $\mathrm{SO}(2,2) = \mathrm{SL}(2,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})_{\mathrm{IIB}}$, and therefore will be collectively denoted by Φ^λ_i, where $\lambda = (\alpha,\, a) = 1,\,2,\,3,\,4$ labels the $\mathbf{4}$ of $\mathrm{SO}(2,2)$, with a choice of basis corresponding to the invariant metric $\eta_{\lambda\sigma} = \mathrm{diag}(+1,\,+1,\,-1,\,-1)$. Its expression in terms of the fields B_{ia} and C_{ia} is

$$\Phi^\lambda_i = \frac{1}{\sqrt{2}}\{C_{i2} - B_{i1},\, B_{i2} + C_{i1},\, B_{i1} + C_{i2},\, -B_{i2} + C_{i1}\}\,. \tag{3.18}$$

We shall use the same notation for the corresponding generators, $\{T^i_\lambda\} \equiv \{T^{1ia},\, T^{2ia}\}$.

[4]In our conventions β_1 is the end root of the long leg and β_5, β_6 the symmetric roots.

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GL(4) × GL(2)-rep.	generator	root	field	dim.
–	$T_{(0,0,0)}$	$\{\epsilon_i - \epsilon_j,\, \epsilon_a - \epsilon_b\}$ $(i<j,\, a>b)$	$\{g_{ij},\, g_{ab}\}$	7
$(\mathbf{1},\mathbf{1})_{(0,0,1)}$	T_0	a	C_0	1
$(\mathbf{4},\mathbf{2})_{(0,1,0)}$	T^{1ia}	$\epsilon_i + \epsilon_a$	B_{ia}	8
$(\mathbf{4},\mathbf{2})_{(0,1,1)}$	T^{2ia}	$a + \epsilon_i + \epsilon_a$	C_{ia}	8
$(\mathbf{6},\mathbf{1})_{(0,2,1)}$	T^{ij}	$a + (\epsilon_i + \epsilon_a) + (\epsilon_j + \epsilon_b)$	$C_{ij\,ab} \equiv C_{ij}\,\epsilon_{ab}$	6
$(\mathbf{1},\mathbf{1})_{(2,0,0)}$	T	$\beta = a + \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4$	$C_{ijkl} \equiv c$	1

Table 5: Axionic fields for the $T_4 \times T_2$ IIB orientifold, generators of Solv(so(6,6)), $\mathrm{O}(1,1)^3$ gradings, and $\mathrm{SL}(4,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ representations.

From the assigned gradings one can conclude that the generators T_0, T^i_λ and T^{ij} close a 23-dimensional nilpotent solvable subalgebra N_7 of Solv(so(6,6)). The non-trivial commutation relations are determined by the grading and the index structure of the generators, and read

$$\begin{aligned} \left[T_0,\, T^i_\lambda\right] &= M_\lambda{}^{\lambda'}\, T^i_{\lambda'}\,, \\ \left[T^i_\lambda,\, T^j_{\lambda'}\right] &= \eta_{\lambda\lambda'}\, T^{ij}\,, \end{aligned} \tag{3.19}$$

where $M_\lambda{}^{\lambda'}$ is a nilpotent generator acting on the $\mathbf{4}$ of SO(2,2) which, for our choice of basis, can be cast in the form

$$M_\lambda{}^{\lambda'} = \frac{1}{2} \begin{pmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{pmatrix}. \tag{3.20}$$

Infinitesimal transformations. Let us consider now the infinitesimal transformations of the scalar fields generated by T_0, $T_{\lambda i}$ and T_{ij}. For simplicity we shall restrict our analysis to those points in the moduli space where the only non-vanishing scalars are Φ^λ_i, C^{ij} and C. The corresponding coset representative thus takes the simple form

$$L = \exp\left(C_{ij}\, T^{ij}\right)\, \exp\left(\Phi^\lambda_i\, T^i_\lambda\right)\, \exp\left(C\, T_0\right), \tag{3.21}$$

and its associated left-invariant one-form is

$$\begin{aligned} L^{-1}dL &= (L^{-1}\partial_0 L)\, dC + (L^{-1}\partial^i_\lambda L)\, d\Phi^\lambda_i + (L^{-1}\partial^{ij} L)\, dC_{ij} \\ &= T_0\, dC + d\Phi^\lambda_i\, (\delta^{\lambda'}_\lambda - C M_\lambda{}^{\lambda'})\, T^i_{\lambda'} + \frac{1}{2}\, T^{ij}\, d\Phi^\lambda_i \Phi_{\lambda j} + T^{ij}\, dC_{ij}\,. \end{aligned} \tag{3.22}$$

In general, the action of an element T_Λ on the coset representative can be expressed as:

$$L^{-1} T_\Lambda L = k^\alpha_\Lambda L^{-1}\partial_\alpha L\,, \tag{3.23}$$

where the k_Λ are the corresponding Killing vectors. In the case at hand, from eq. (3.19), we can derive

$$\begin{aligned} L^{-1}T_0 L &= T_0 + \Phi^\lambda_i\, M_\lambda{}^{\lambda'}\, T^i_{\lambda'} + \tfrac{1}{2}\, \Phi^\lambda_i\, \Phi^{\lambda'}_j\, M_{\lambda\lambda'}\, T^{ij}\,, \\ L^{-1}T^i_\lambda L &= \left(\delta^{\lambda'}_\lambda - C M_\lambda{}^{\lambda'}\right)\, T^i_{\lambda'} + T^{ij}\, \Phi_{j\lambda}\,, \\ L^{-1}T^{ij} L &= T^{ij}\,, \end{aligned} \tag{3.24}$$

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and, thus, read the non-vanishing components of the Killing vectors

$$
\begin{aligned}
k_0 &= \partial_0 + \Phi_i^\lambda\, M_\lambda{}^{\lambda'}\, \partial^i_{\lambda'}\,, \\
k^i_\lambda &= \partial^i_\lambda + \frac{1}{2}\,\Phi_{j\lambda}\,\partial^{ij}\,, \\
k^{ij} &= \partial^{ij}\,,
\end{aligned}
\tag{3.25}
$$

where

$$
\partial_0 = \frac{\partial}{\partial C}\,, \qquad \partial^{ij} = \frac{\partial}{\partial C_{ij}} \qquad \text{and} \qquad \partial^i_\lambda = \frac{\partial}{\partial \Phi^\lambda_i}\,. \tag{3.26}
$$

Therefore, under the infinitesimal diffeomorphism $\xi^0 k_0 + \xi^\lambda_i k^i_\lambda + \xi_{ij} k^{ij}$ the fields transform as follows:

$$
\begin{aligned}
\delta C &= \xi^0\,, \\
\delta \Phi^\lambda_i &= \xi^\lambda_i + \xi^0\, \Phi^{\lambda'}\, M_{\lambda'}{}^\lambda\,, \\
\delta C_{ij} &= \xi_{ij} + \frac{1}{2}\,\xi^\lambda_{[i}\,\Phi_{j]\lambda}\,.
\end{aligned}
\tag{3.27}
$$

Scalar kinetic terms. Since all the quantities of our gauging are covariant with respect to $\mathrm{SO}(2,2)\times\mathrm{GL}(4,\mathbb{R})$ it is useful to define the (full) coset representative in the following way

$$
L = \exp\left(C_{ij}\,T^{ij}\right)\,\exp\left(\Phi^\lambda_i\,T^i_\lambda\right)\,\exp\left(c\,T\right)\mathbb{E}, \tag{3.28}
$$

where $\mathbb{E}$ is the coset representative of the submanifold

$$
\mathbb{E} \in \mathrm{O}(1,1)_0 \times \frac{\mathrm{SO}(2,2)}{\mathrm{SO}(2)\times\mathrm{SO}(2)} \times \frac{\mathrm{GL}(4,\mathbb{R})}{\mathrm{SO}(4)}\,. \tag{3.29}
$$

The scalar kinetic terms are computed by evaluating the components of the vielbein $\mathscr{P} = L^{-1}dL_{|G/H}$:

$$
L^{-1}dL_{|G/H} = \mathscr{P}_{\hat\imath\hat\jmath}\,\hat T^{\hat\imath\hat\jmath} + \mathscr{P}^{\hat\lambda}_{\hat\imath}\,\hat T^{\hat\imath}_{\hat\lambda} + \mathscr{P}\,\hat T + \mathscr{P}_E\,, \tag{3.30}
$$

where the restriction to G/H amounts to select the non-compact isometries of the scalar manifold, $\mathscr{P}_E$ is the algebra-valued vielbein of the submanifold (3.29). Finally, the hatted generators denote the non-compact component of the corresponding solvable generator. The kinetic lagrangian for the scalar fields is then

$$
\mathscr{L}_{\mathrm{scal}} = \frac{1}{2}\,\mathscr{P}_\mu\,\mathscr{P}^\mu + \frac{1}{2}\sum_{\hat\imath\hat\lambda}\,\mathscr{P}^{\hat\lambda}_{\hat\imath\,\mu}\,\mathscr{P}^{\hat\lambda\,\mu}_{\hat\imath} + \frac{1}{4}\sum_{\hat\imath\hat\jmath}\,\mathscr{P}_{\hat\imath\hat\jmath\,\mu}\,\mathscr{P}_{\hat\imath\hat\jmath}{}^\mu + \mathrm{Tr}(\mathscr{P}_E^2)\,, \tag{3.31}
$$

where

$$
\begin{aligned}
\mathscr{P}_\mu &= \partial_\mu c\,, \\
\mathscr{P}^{\hat\lambda}_{\hat\imath\mu} &= (\partial_\mu \Phi^\lambda_i)\,E^i{}_{\hat\imath}\,E_\lambda{}^{\hat\lambda}\,, \\
\mathscr{P}_{\hat\imath\hat\jmath\mu} &= \left[\partial_\mu C_{ij} + \frac{1}{4}\left(\partial_\mu \Phi^\lambda_i\,\Phi_{j\lambda} - \partial_\mu \Phi^\lambda_j\,\Phi_{i\lambda}\right)\right]\,E^i{}_{\hat\imath}\,E^j{}_{\hat\jmath}\,.
\end{aligned}
\tag{3.32}
$$

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Sp-section	$O(1,1)^3$-grading	weight
F_{1a}	$(-1,0,-1/2)$	$\epsilon_a - 1/\sqrt{2}\epsilon_7$
F_{2a}	$(-1,0,1/2)$	$w + \epsilon_a$
$\mathscr{F}^i$	$(-1,-1,-1/2)$	$-\epsilon_i - 1/\sqrt{2}\epsilon_7$
F^i	$(1,-1,-1/2)$	$w + \epsilon_j + \epsilon_k + \epsilon_l$
$\tilde{F}^{1a}$	$(1,0,1/2)$	$-\epsilon_a + 1/\sqrt{2}\epsilon_7$
$\tilde{F}^{2a}$	$(1,0,-1/2)$	$-w - \epsilon_a$
$\tilde{\mathscr{F}}_i$	$(1,1,1/2)$	$\epsilon_i + 1/\sqrt{2}\epsilon_7$
$\tilde{F}_i$	$(-1,1,1/2)$	$-w - \epsilon_j - \epsilon_k - \epsilon_l$

Table 6: Field strengths, $O(1,1)^3$ gradings, and corresponding weights.

Vector fields. The twelve vector potentials are $B_{a\mu}$, $C_{a\mu}$, G^i_μ, $C_{ijk\mu}$. As before, we shall collectively denote by A^λ_μ the pair $\{B_{a\mu}, C_{a\mu}\}$, and by $F^\lambda = dA^\lambda$ the corresponding field strengths. To avoid confusion, we shall then adopt the following notation for the remaining field strengths: $\mathscr{F}^i = dG^i$ and $F^i = \epsilon^{ijkl}\, dC_{jkl}$. Moreover, $\tilde{F}_\lambda$, $\tilde{\mathscr{F}}_i$ and $\tilde{F}_i$ will denote the "dual" field strengths, obtained by varying the lagrangian with respect to the electric ones, not to be confused with the four-dimensional Hodge duals ${}^*F^\lambda$, ${}^*\mathscr{F}^i$ and ${}^*F^i$. Following [37], we can then collect the field strengths and their duals in a symplectic vector

$$\{F^\lambda,\, \mathscr{F}^i,\, F^i,\, \tilde{F}_\lambda,\, \tilde{\mathscr{F}}_i,\, \tilde{F}_i\}\,. \tag{3.33}$$

In table 6, we list the field strengths and their duals as they appear in the symplectic section, together with their $O(1,1)^3$ gradings and the corresponding weights of the **56** of $E_{7(7)}$.

Under a generic nilpotent transformation

$$\xi T + \xi_0\, T_0 + \xi^\lambda_i T^i_\lambda + \xi_{ij} T^{ij}\,, \tag{3.34}$$

the field strengths transform as

$$\begin{aligned}
\delta F^\lambda &= -\xi^\lambda_i\, \mathscr{F}^i + \xi_0\, F^{\lambda'}\, M_{\lambda'}{}^\lambda\,,\\
\delta \mathscr{F}^i &= 0\,,\\
\delta F^i &= \xi\, \mathscr{F}^i\,,\\
\delta \tilde{F}_\lambda &= \xi\, \eta_{\lambda\lambda'}\, F^{\lambda'} - \xi_0\, M_\lambda{}^{\lambda'}\, \tilde{F}_{\lambda'} - \eta_{\lambda\lambda'}\, \xi^{\lambda'}_i\, F^i\,,\\
\delta \tilde{\mathscr{F}}_i &= -\xi\, \tilde{F}_i + \xi^\lambda_i\, \tilde{F}_\lambda - 2\,\xi_{ij}\, F^j\,,\\
\delta \tilde{F}_i &= -\xi^{\lambda'}_i\, F^\lambda\, \eta_{\lambda\lambda'} + 2\,\xi_{ij}\, \mathscr{F}^j\,.
\end{aligned} \tag{3.35}$$

We then deduce that the electric subalgebra is

$$g_e = o(1,1)_{(0,0)} + so(2,2)_{(0,0)} + gl(4,\mathbb{R})_{(0,0)} + (\mathbf{1},\mathbf{1})_{(2,0)} + (\mathbf{4},\mathbf{4})_{(0,1)} + (\mathbf{1},\mathbf{6})_{(0,2)}\,,$$

where $o(1,1)_{(0,0)}$ is the generator of $O(1,1)_0$, and the grading refers to $O(1,1)_0 \times O(1,1)_1$. The group $O(1,1)_2$ is now included inside $SO(2,2)$ and, in what follows, we shall not consider its grading any longer. Furthermore, we identify T as the generator in $(\mathbf{1},\mathbf{1})_{(2,0)}$,

T^i_λ and T^{ij} are associated to $(\mathbf{4},\mathbf{4})_{(0,1)}$ and $(\mathbf{6},\mathbf{1})_{(0,2)}$, respectively. The interested reader may find in appendix the explicit symplectic realisation of the generators of N_7, as well as the computation of the vector kinetic matrix.

3.2 $T_2 \times T_4$ IIB orientifold with D5 branes. The ungauged version

Solvable algebra of global symmetries. In this second model the relevant axions are B_{ia}, C_{ab}, $C_{iabc} \equiv C^d_i$, $C_{\mu\nu} \equiv c$ and $C_{ij} = \epsilon_{ij}\, c'$, and can be associated to the following choice of simple roots

$$\begin{aligned}
\beta_1 &= \epsilon_1 - \epsilon_2\,,\\
\beta_2 &= \epsilon_2 + \epsilon_3\,,\\
\beta_3 &= -\epsilon_3 + \epsilon_4\,,\\
\beta_4 &= -\epsilon_4 + \epsilon_5\,,\\
\beta_5 &= -\epsilon_5 + \epsilon_6\,,\\
\beta_6 &= a + \epsilon_3 + \epsilon_4\,,
\end{aligned}$$

for the subalgebra $\mathrm{so}(6,6) \subset e_{7(7)}$. The Dynkin diagram of the external $\mathrm{SL}(2,\mathbb{R})$ consists, instead, of the single root

$$\beta = a + \epsilon_1 + \epsilon_2\,, \tag{3.36}$$

whose corresponding axion is C_{ij}, according to eq. (3.3).

The triple grading, this time, refers to the $\mathrm{O}(1,1)^3$ group generated by the three Cartan H_β, H_{λ^2}, H_{λ^6} and parametrised by the moduli $\beta\cdot h$, h_2, h_6:

$$\begin{aligned}
\mathrm{O}(1,1)_0 &\to e^{\beta\cdot h} &&= V_2\, e^{-\phi/2}\,,\\
\mathrm{O}(1,1)_1 &\to e^{h_2} &&= (V_2)^{1/2}\,(V_4)^{1/4}\, e^{\phi/2}\,,\\
\mathrm{O}(1,1)_2 &\to e^{h_6} &&= (V_4)^{1/2}\, e^{-\phi/2}\,,
\end{aligned} \tag{3.37}$$

where, as usual, $\mathrm{O}(1,1)_0$ is in the external $\mathrm{SL}(2,\mathbb{R})$, while $\mathrm{O}(1,1)_1$ and $\mathrm{O}(1,1)_2$ are contained in $\mathrm{GL}(2,\mathbb{R}) \times \mathrm{GL}(4,\mathbb{R})$.

In table 7 we list the axionic fields of this model, together with the corresponding generator of $\mathrm{Solv}(\mathrm{so}(6,6))$, for each of which the $\mathrm{O}(1,1)^3$ grading is specified, as well as their $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(4,\mathbb{R})$ representations

Also in this case, the generators T, T^{ia}, T^i_a and T^{ab} close a 23-dimensional solvable subalgebra of $\mathrm{SO}(6,6)$

$$N_5 = c\,T + B_{ia}\,T^{ia} + C^a_i\,T^i_a + C_{ab}\,T^{ab}\,, \tag{3.38}$$

whose algebraic structure is encoded in the non-vanishing commutators

$$\left[T^{ia},\, T^{bc}\right] = \epsilon^{abcd}\,T^i_d\,, \tag{3.39}$$

$$\left[T^{ia},\, T^j_d\right] = \epsilon^{ij}\,\delta^a_d\,T\,. \tag{3.40}$$

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GL(2) × GL(6)-rep.	generator	root	field	dim.
–	$T_{(0,0,0)}$	$\{\epsilon_i - \epsilon_j,\, \epsilon_a - \epsilon_b\}$ $(i<j,\, a>b)$	$\{g_{ij},\, g_{ab}\}$	7
$(\mathbf{1},\mathbf{6})_{(0,0,1)}$	T^{ab}	$\alpha_7 + \epsilon_a + \epsilon_b$	C_{ab}	6
$(\mathbf{2},\mathbf{4})_{(0,1,0)}$	T^{ia}	$\epsilon_i + \epsilon_a$	B_{ia}	8
$(\mathbf{2},\overline{\mathbf{4}})_{(0,1,1)}$	T^i_d	$\alpha_7 + \epsilon_i + \epsilon_a + \epsilon_b + \epsilon_c$	C^d_i	8
$(\mathbf{1},\mathbf{1})_{(0,2,1)}$	T	$\alpha_7 + \epsilon_i + \epsilon_j + \epsilon_a + \epsilon_b + \epsilon_c + \epsilon_d$	$C_{\mu\nu} \;=\; c$	1
$(\mathbf{1},\mathbf{1})_{(2,0,0)}$	T'	β	$C_{ij} \;=\; c'$	1

Table 7: Axionic fields for the $T_2 \times T_4$ IIB orientifold, generators of Solv(so(6, 6)), $O(1,1)^3$ gradings, and $SL(2,\mathbb{R}) \times SL(4,\mathbb{R})$ representations.

The corresponding coset representative reads

$$L = e^{c\,T}\, e^{B_{ia}\,T^{ia}}\, e^{C^a_i\,T^i_a}\, e^{C_{ab}\,T^{ab}}\,, \tag{3.41}$$

while its left-invariant one-form is

$$L^{-1}dL = T dc + T^{ab} dC_{ab} + T^i_d\, dC^d_i + (T^{ia} + \epsilon^{ij}\, C^a_j\, T + \epsilon^{abcd}\, T^i_d\, C_{bc})\, dB_{ia}\,. \tag{3.42}$$

The transformation properties of the axionic scalars can be deduced from

$$\begin{aligned}
L^{-1} T L &= T\,,\\
L^{-1} T^i_a L &= T^i_a + \epsilon^{ij}\, B_{ja}\, T\,,\\
L^{-1} T^{ia} L &= T^{ia} + \epsilon^{ij}\, C^a_j\, T + \epsilon^{abcd}\, T^i_d\, C_{bc}\,,\\
L^{-1} T^{ab} L &= T^{ab} + \epsilon^{abcd}\, B_{id}\, T^i_c\,,
\end{aligned} \tag{3.43}$$

which identify the Killing vectors

$$\begin{aligned}
k &= \partial\,,\\
k^i_a &= \partial^i_a + \epsilon^{ij} B_{ja} \partial\,,\\
k^{ia} &= \partial^{ia}\,,\\
k^{ab} &= \partial^{ab} + \epsilon^{abcd} B_{id} \partial^i_c\,,
\end{aligned} \tag{3.44}$$

where

$$\partial = \frac{\partial}{\partial c}\,, \qquad \partial^i_a = \frac{\partial}{\partial C^a_i}\,, \qquad \partial^{ia} = \frac{\partial}{\partial B_{ia}}\,, \qquad \partial^{ab} = \frac{\partial}{\partial C_{ab}}\,. \tag{3.45}$$

Hence, under the infinitesimal diffeomorphism $\xi\, T + \xi_{ia}\, T^{ia} + \xi^a_i\, T^i_a + \xi_{ab}\, T^{ab}$, one has

$$\begin{aligned}
\delta c &= \epsilon^{ij}\, \xi^a_i\, B_{ja} + \xi\,,\\
\delta C^a_i &= \epsilon^{abcd}\, \xi_{bc}\, B_{id} + \xi^a_i\,,\\
\delta B_{ia} &= \xi_{ia}\,,\\
\delta C_{ab} &= \xi_{ab}\,.
\end{aligned} \tag{3.46}$$

For later convenience we shall define the generator $T_{ab} = -\frac{1}{4}\,\epsilon_{abcd}\, T^{cd}$, and the corresponding parameter $\xi_{ab} = -\frac{1}{4}\,\epsilon_{abcd}\, \xi^{cd}$, in terms of which the relation (3.39) reads

$$[T_{ab},\, T^{ic}] = \delta^c_{[a}\, T^i_{b]}\,. \tag{3.47}$$

Sp-section	$O(1,1)^3$-grading	weight
$\mathscr{F}^i_{\mu\nu}$	$(-1,-1,-1/2)$	$-\epsilon_i - 1/\sqrt{2}\epsilon_7$
$F_{i\mu\nu}$	$(1,-1,-1/2)$	$w+\epsilon_i$
$\mathscr{H}_{a\mu\nu}$	$(-1,0,-1/2)$	$\epsilon_a - 1/\sqrt{2}\epsilon_7$
$F^a_{\mu\nu}$	$(-1,0,1/2)$	$w+\epsilon_b+\epsilon_c+\epsilon_d$
$\tilde{\mathscr{F}}_{i\mu\nu}$	$(1,1,1/2)$	$\epsilon_i + 1/\sqrt{2}\epsilon_7$
$\tilde{F}^i_{\mu\nu}$	$(-1,1,1/2)$	$w+\epsilon_j+\epsilon_a+\epsilon_b+\epsilon_c+\epsilon_d$
$\tilde{\mathscr{H}}^a_{\mu\nu}$	$(1,0,1/2)$	$-\epsilon_a + 1/\sqrt{2}\epsilon_7$
$\tilde{F}_{a\mu\nu}$	$(1,0,-1/2)$	$w+\epsilon_i+\epsilon_j+\epsilon_a$

Table 8: Field strengths, $O(1,1)^3$ gradings, and corresponding weights.

Vector fields. The vector fields of this model are G^i_μ, $C_{i\mu}\, B_{a\mu}$, C^a_μ, and we name the corresponding field strengths and their duals by

$$\mathscr{F}^i_{\mu\nu},\, F_{i\mu\nu},\, \mathscr{H}_{a\mu\nu},\, F^a_{\mu\nu},\, \tilde{\mathscr{F}}_{i\mu\nu},\, \tilde{F}^i_{\mu\nu},\, \tilde{\mathscr{H}}^a_{\mu\nu},\, \tilde{F}_{a\mu\nu}\,. \tag{3.48}$$

In the table 8 we list the field strengths and their duals as they appear in the symplectic section, together with their $O(1,1)^3$ gradings, and the corresponding $E_{7(7)}$ weights.

The transformation laws under a generic nilpotent transformation $\xi'\, T' + \xi\, T + \xi^{ab}\, T_{ab} + \xi_{ia}\, T^{ia} + \xi^a_i\, T^i_a$ can be deduced from the grading and weight structures. One finds

$$\begin{aligned}
\delta\mathscr{F}^i &= 0\,,\\
\delta F_i &= \xi'\,\epsilon_{ij}\,\mathscr{F}^j\,,\\
\delta\mathscr{H}_a &= \xi_{ia}\,\mathscr{F}^i\,,\\
\delta F^a &= \xi^{ab}\,\mathscr{H}_b - \xi^a_i\,\mathscr{F}^i\,,\\
\delta\tilde{\mathscr{F}}_i &= \xi'\,\epsilon_{ij}\,\tilde{F}^j + \xi^a_i\,\tilde{F}_a - \xi_{ia}\,\tilde{\mathscr{H}}^a + \xi\, F_i\,,\\
\delta\tilde{F}^i &= \epsilon^{ij}\,\xi_{ja}\,F^a - \epsilon^{ij}\,\xi^a_j\,\mathscr{H}_a + \xi\,\mathscr{F}^i\\
\delta\tilde{\mathscr{H}}^a &= \xi'\,F^a + \xi^{ab}\,\tilde{F}_b + \xi^a_i\,\epsilon^{ij}\,F_j\,,\\
\delta\tilde{F}_a &= \xi'\,\mathscr{H}_a - \xi_{ai}\,\epsilon^{ij}\,F_j\,.
\end{aligned} \tag{3.49}$$

The explicit symplectic representation of the N_5 generators together with the computation of the vector kinetic matrix $\mathscr{N}$ may be found in appendix.

3.3 $T_0 \times T_6$ and $T_6 \times T_0$ IIB orientifolds with D3 and D9 branes. The ungauged version

The $T_0 \times T_6$ model in the presence of $D3$-branes, with and without fluxes was constructed in [30]–[32]. The structure of the $T_6 \times T_0$ model, on the other hand, is somewhat trivial, since there is no room for fluxes to be turned on. For completeness, here we shall confine ourselves to the description of their embeddings within the $\mathscr{N} = 8$ theory, and to the identification of the solvable algebras N_3 and N_9, together with their action on scalar and vector fields.

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GL(6)-rep.	generator	root	field	dim.
–	$T_{(0,0)}$	$\{\epsilon_a - \epsilon_b\}$ $(a > b)$	$\{g_{ab}\}$	15
$\overline{\mathbf{15}}_{(0,1)}$	T_{ab}	$a + \epsilon_c + \epsilon_d + \epsilon_e + \epsilon_f$	$C^{ab} \equiv \epsilon^{abcdef} C_{cdef}$	15
$\mathbf{1}_{(2,0)}$	T	β	$C_0 = c$	1

Table 9: Axionic fields for the $T_0 \times T_6$ IIB orientifold, generators of Solv(so(6,6)), $\mathrm{O}(1,1)^2$ gradings and $\mathrm{GL}(6,\mathbb{R})$ representations.

Solvable algebra of global symmetries: the $T_0 \times T_6$ model. The embedding of the $\mathrm{sl}(2,\mathbb{R}) + \mathrm{so}(6,6)$ algebra inside $e_{7(7)}$ is defined by the following identification of the simple roots:

$$\begin{aligned} \beta_1 &= -\epsilon_1 + \epsilon_2 \,, \\ \beta_2 &= -\epsilon_2 + \epsilon_3 \,, \\ \beta_3 &= -\epsilon_3 + \epsilon_4 \,, \\ \beta_4 &= -\epsilon_4 + \epsilon_5 \,, \\ \beta_5 &= -\epsilon_5 + \epsilon_6 \,, \\ \beta_6 &= a + \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 \,, \end{aligned} \tag{3.50}$$

for the so(6,6) component, and

$$\beta = a \,, \tag{3.51}$$

for the $\mathrm{sl}(2,\mathbb{R})$ one. The correspondence axion-root is quite simple and is summarised in table 9.

In this case, the grading is with respect to the pair of O(1,1) groups generated by H_β, H_{λ^6} and corresponding to the following moduli:

$$\begin{aligned} \mathrm{O}(1,1)_0 &\rightarrow e^{\beta\cdot h} = e^{-\phi} \,, \\ \mathrm{O}(1,1)_1 &\rightarrow e^{\lambda^6\cdot h} = V_6 \,. \end{aligned} \tag{3.52}$$

Sp-section	$\mathrm{O}(1,1)^2$-grading	weight
$F_{a\mu\nu}$	$(1,-1/2)$	$w + \epsilon_a$
$\mathscr{H}_{a\mu\nu}$	$(-1,-1/2)$	$\epsilon_a - 1/\sqrt{2}\epsilon_7$
$\tilde{F}^a_{\mu\nu}$	$(-1,1/2)$	$-w - \epsilon_a$
$\tilde{\mathscr{H}}^a_{\mu\nu}$	$(1,1/2)$	$-\epsilon_a + 1/\sqrt{2}\epsilon_7$

Table 10: Field strengths, $\mathrm{O}(1,1)^2$ gradings, and corresponding weights.

The nilpotent algebra N_3, generated by T_{ab}, acts as Peccei-Quinn translations on the R-R scalars C^{ab}

$$\delta C^{ab} = \xi^{ab} \,. \tag{3.53}$$

The vector fields are $C_{a\mu}$ and $B_{a\mu}$, and the symplectic section of the corresponding field strengths $F_{a\mu\nu}$ and $\mathscr{H}_{a\mu\nu}$ and their magnetic duals $\tilde{F}^a_{\mu\nu}$, $\tilde{\mathscr{H}}^a_{\mu\nu}$ is listed in table 10.

The duality action of an infinitesimal transformation $\xi^{ab}\, T_{ab} + \xi\, T$ is then

$$\begin{aligned} \delta F_a &= \xi\, \mathscr{H}_a \,, \\ \delta \mathscr{H}_a &= 0 \,, \\ \delta \tilde{F}^a &= \xi^{ab}\, \mathscr{H}_b \,, \\ \delta \tilde{\mathscr{H}}^a &= -\xi^{ab}\, F_b - \xi\, \tilde{F}^a \,. \end{aligned} \tag{3.54}$$

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GL(6)-rep.	generator	root	field	dim.
–	$T_{(0,0)}$	$\{\epsilon_i - \epsilon_j\}$ $(i<j)$	$\{g_{ij}\}$	15
$\mathbf{15}_{(0,1)}$	T^{ij}	$a+\epsilon_i+\epsilon_j$	C_{ij}	15
$\mathbf{1}_{(2,0)}$	T	β	$C_{\mu\nu} = c$	1

Table 11: Axionic fields for the $T_6 \times T_0$ IIB orientifold, generators of Solv(so(6,6)), $\mathrm{O}(1,1)^2$ gradings, and $\mathrm{GL}(6,\mathbb{R})$ representations.

Solvable algebra of global symmetries: the $T_6 \times T_0$ model. The embedding of the $\mathrm{sl}(2,\mathbb{R}) + \mathrm{so}(6,6)$ algebra inside $e_{7(7)}$ is defined by the following identification of the simple roots

$$\begin{aligned}
\beta_1 &= \epsilon_1 - \epsilon_2\,,\\
\beta_2 &= \epsilon_2 - \epsilon_3\,,\\
\beta_3 &= \epsilon_3 - \epsilon_4\\
\beta_4 &= \epsilon_4 - \epsilon_5\,,\\
\beta_5 &= \epsilon_5 - \epsilon_6\,,\\
\beta_6 &= a + \epsilon_5 + \epsilon_6\,,
\end{aligned} \tag{3.55}$$

for the so(6,6) component, and

$$\beta = a + \sum_{n=1}^{6} \epsilon_n\,, \tag{3.56}$$

for the $\mathrm{sl}(2,\mathbb{R})$ one. The correspondence axion-root is quite simple, and is summarised in table 11.

In this case, the grading is with respect to a pair of O(1,1) groups generated by H_β, H_{λ^6} and corresponding to the following moduli:

$$\begin{aligned}
\mathrm{O}(1,1)_0 &\to e^{\beta\cdot h} = V_6\, e^{\phi/2}\,,\\
\mathrm{O}(1,1)_1 &\to e^{\lambda^6\cdot h} = (V_6)^{1/2}\, e^{-3/4\,\phi}\,.
\end{aligned}$$

Sp-section	$\mathrm{O}(1,1)^2$-grading	weight
$F_{i\mu\nu}$	$(-1,1/2)$	$w+\epsilon_i$
$\mathscr{F}^i_{\mu\nu}$	$(-1,-1/2)$	$-\epsilon_i - 1/\sqrt{2}\epsilon_7$
$\tilde{F}^i_{\mu\nu}$	$(1,-1/2)$	$-w-\epsilon_i$
$\tilde{\mathscr{F}}_{i\mu\nu}$	$(1,1/2)$	$\epsilon_i + 1/\sqrt{2}\epsilon_7$

Table 12: Field strengths, $\mathrm{O}(1,1)^2$ gradings, and corresponding weights.

The nilpotent algebra N_9, generated by T^{ij}, acts as Peccei-Quinn translations on the R-R scalars C_{ij},

$$\delta C_{ij} = \xi_{ij}\,. \tag{3.57}$$

The vector fields are $C_{i\mu}$ and G^i_μ, and the symplectic sections of the corresponding field strengths $F_{i\mu\nu}$ and $\mathscr{F}^i_{\mu\nu}$ and their magnetic duals $\tilde{F}^i_{\mu\nu}$, $\tilde{\mathscr{F}}_{i\mu\nu}$ are listed in table 12.

The duality action of an infinitesimal transformation $\xi_{ij}\, T^{ij} + \xi\, T$ is then

$$\begin{aligned}
\delta F_i &= \xi_{ij}\, \mathscr{F}^j\,,\\
\delta \mathscr{F}^i &= 0\,,\\
\delta \tilde{F}^i &= \xi\, \mathscr{F}^i\,,\\
\delta \tilde{\mathscr{F}}_i &= \xi_{ij}\, \tilde{F}^j + \xi\, F_i\,.
\end{aligned} \tag{3.58}$$

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GL(5)-rep.	generator	root	field	dim.
—	$T_{(0,0,0)}$	$\{\epsilon_a - \epsilon_b\}$ $(a > b)$	$\{g_{ab}\}$	10
$\overline{\mathbf{10}}_{(0,0,1)}$	T_{ab}	$a + \epsilon_c + \epsilon_d + \epsilon_e$	$C_{cde} \equiv C^{ab}$	10
$\mathbf{5}_{(0,1,0)}$	T^a	$\epsilon_1 + \epsilon_a$	$B_{1a} \equiv B_a$	5
$\overline{\mathbf{5}}_{(0,1,1)}$	T_e	$a + \epsilon_1 + \epsilon_a + \epsilon_b + \epsilon_c + \epsilon_d$	$C_{\mu\nu a} \equiv C^e$	5
$\mathbf{1}_{(2,0,0)}$	T	β	$C_1 = c$	1

Table 13: Axionic fields for the $T_1 \times T_5$ IIA orientifold, generators of Solv(so(6, 6)), $O(1,1)^3$ gradings, and $GL(5,\mathbb{R})$ representations.

As a result, the electric group contains the whole SO(6, 6), as for the heterotic string on T_6. In other words, there are no Peccei-Quinn isometries in SO(6, 6) which could be gauged. This feature is consistent with the fact that this model does not allow fluxes, and usually fluxes translate into local Peccei-Quinn invariances in the low-energy supergravity description.

3.4 $T_1 \times T_5$ IIA orientifold with $D4$-branes

Solvable algebra of global symmetries. The embedding of the $sl(2,\mathbb{R}) + so(6,6)$ algebra inside $e_{7(7)}$ is defined by the following identifications of simple roots:

$$
\begin{aligned}
\beta_1 &= \epsilon_1 + \epsilon_2 \,, \\
\beta_2 &= -\epsilon_2 + \epsilon_3 \,, \\
\beta_3 &= -\epsilon_3 + \epsilon_4 \,, \\
\beta_4 &= -\epsilon_4 + \epsilon_5 \,, \\
\beta_5 &= -\epsilon_5 + \epsilon_6 \,, \\
\beta_6 &= a + \epsilon_2 + \epsilon_3 + \epsilon_4 \,,
\end{aligned}
\tag{3.59}
$$

for the so(6, 6) factor, and

$$\beta = a + \epsilon_1 \,, \tag{3.60}$$

for the $sl(2,\mathbb{R})$ one. The correspondence axion-root is quite simple, and is summarised in table 13.

In this case the grading is with respect to the $O(1,1)^3$ group generated by H_β, H_{λ^1}, H_{λ^6} and parametrised by the moduli $\beta \cdot h$, h_1, h_6:

$$
\begin{aligned}
O(1,1)_0 &\to e^{\beta\cdot h} = V_1\, e^{-3\phi/4} \,, \\
O(1,1)_1 &\to e^{h_1} = V_1\, (V_5)^{1/5}\, e^{\phi/2} \,, \\
O(1,1)_2 &\to e^{h_6} = (V_5)^{3/2}\, e^{-\phi/4} \,.
\end{aligned}
\tag{3.61}
$$

The generators T^a, T_a and T_{ab} close a twenty-dimensional nilpotent subalgebra N_4 of Solv(so(6, 6)):

$$N_4 = B_a\, T^a + C^a\, T_a + C^{ab}\, T_{ab} \,, \tag{3.62}$$

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vector	$\mathrm{O}(1,1)^3$-grading	weight
$F_{\mu\nu}$	$(1,-1,-1/2)$	w
$\mathscr{F}^1_{\mu\nu}$	$(-1,-1,-1/2)$	$-\epsilon_1 - 1/\sqrt{2}\epsilon_7$
$F_{1a\mu\nu}$	$(1,0,-1/2)$	$w+\epsilon_1+\epsilon_a$
$\mathscr{H}_{a\mu\nu}$	$(-1,0,-1/2)$	$\epsilon_a - 1/\sqrt{2}\epsilon_7$
$\tilde{F}_{\mu\nu}$	$(-1,1,1/2)$	$-w$
$\tilde{\mathscr{F}}_{1\mu\nu}$	$(1,1,1/2)$	$\epsilon_1 + 1/\sqrt{2}\epsilon_7$
$\tilde{F}^{1a}_{\mu\nu}$	$(-1,0,1/2)$	$-w-\epsilon_1-\epsilon_a$
$\tilde{\mathscr{H}}^a_{\mu\nu}$	$(1,0,1/2)$	$-\epsilon_a + 1/\sqrt{2}\epsilon_7$

Table 14: Field strengths, $\mathrm{O}(1,1)^3$ gradings, and corresponding weights.

whose algebraic structure is encoded in the non-vanishing commutator

$$[T_{ab},\, T^c] = T_{[a}\delta^c_{b]}\,. \tag{3.63}$$

The corresponding coset representative reads

$$L = e^{C^a\,T_a}\, e^{B_a\,T^a}\, e^{C^{ab}\,T_{ab}}\, e^{c\,T}\,\mathbb{E}\,, \tag{3.64}$$

where the $\mathbb{E}$ factor parametrises the submanifold:

$$\mathrm{O}(1,1)_0 \times \mathrm{O}(1,1)_1 \times \mathrm{O}(1,1)_2 \times \frac{\mathrm{SL}(5,\mathbb{R})}{\mathrm{SO}(5)}\,. \tag{3.65}$$

A generic element $\xi_a\,T^a + \xi^a\,T_a + \xi^{ab}\,T_{ab}$ of N_4 then induces the following transformations on the axionic scalars

$$\begin{aligned} \delta C^a &= \xi^a + \xi^{ab}\,B_b\,, \\ \delta B_a &= \xi_a\,, \\ \delta C^{ab} &= \xi^{ab}\,. \end{aligned} \tag{3.66}$$

Vector fields. The vector fields of this model are C_μ, G^1_μ, $C_{1a\mu}$, $B_{a\mu}$, and we name the corresponding field strengths $F_{\mu\nu}$, $\mathscr{F}^1_{\mu\nu}$, $F_{1a\mu\nu}$, $\mathscr{H}_{a\mu\nu}$. The symplectic section of the field strengths and their duals is

$$\{F_{\mu\nu},\, \mathscr{F}^1_{\mu\nu},\, F_{1a\mu\nu},\, \mathscr{H}_{a\mu\nu},\, \tilde{F}_{\mu\nu},\, \tilde{\mathscr{F}}_{1\mu\nu},\, \tilde{F}^{1a}_{\mu\nu},\, \tilde{\mathscr{H}}^a_{\mu\nu}\}\,, \tag{3.67}$$

and in table 14 we give their $\mathrm{O}(1,1)^3$ gradings and the corresponding $E_{7(7)}$ weights.

The action of infinitesimal duality transformation $\xi_a\,T^a + \xi^a\,T_a + \xi^{ab}\,T_{ab} + \xi\,T$ on the symplectic section is

$$\begin{aligned} \delta F &= \xi\,\mathscr{F}^1\,, \\ \delta \mathscr{F}^1 &= 0\,, \\ \delta F_{1a} &= \xi_a\,F + \xi\,\mathscr{H}_a\,, \end{aligned}$$

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GL(3) × GL(3)-rep.	generator	root	field	dim.
—	$T_{(0,0,0)}$	$\{\epsilon_i - \epsilon_j,\, \epsilon_a - \epsilon_b\}$ $(i<j,\, a>b)$	$\{g_{ij},\, g_{ab}\}$	6
$(\mathbf{1},\mathbf{3})_{(0,0,1)}$	T_{ab}	$a + \epsilon_c$	C^{ab}	3
$(\mathbf{3},\mathbf{3})_{(0,1,0)}$	T^{ia}	$\epsilon_i + \epsilon_a$	B_{ia}	9
$(\mathbf{3},\overline{\mathbf{3}})_{(0,1,1)}$	T^i_a	$a + \epsilon_i + \epsilon_b + \epsilon_c$	$C_{ibc} \equiv C_i^a$	9
$(\overline{\mathbf{3}},\mathbf{1})_{(0,2,1)}$	T^{ij}	$\epsilon_i + \epsilon_j + \epsilon_a + \epsilon_b + \epsilon_c$	$C_{k\mu\nu} \equiv C_{ij}$	3
$(\mathbf{1},\mathbf{1})_{(2,0,0)}$	T	β	$C_{ijk} \equiv c$	1

Table 15: Axionic fields for the $T_3 \times T_3$ IIA orientifold, generators of Solv(so(6,6)), $\mathrm{O}(1,1)^3$ gradings, and $\mathrm{GL}(3,\mathbb{R}) \times \mathrm{GL}(3,\mathbb{R})$ representations.

$$\begin{aligned} \delta\mathscr{H}_a &= \xi_a\, \mathscr{F}^1 \,, \\ \delta\tilde{F} &= -\xi_a\, \tilde{F}^{1a} + \xi^a\, \mathscr{H}_a \,, \\ \delta\tilde{\mathscr{F}}_1 &= -\xi_a\, \tilde{\mathscr{H}}^a - \xi^a\, F_{1a} - \xi\, \tilde{F} \,, \\ \delta\tilde{F}^{1a} &= -\xi^a\, \mathscr{F}^1 - \xi^{ab}\, \mathscr{H}_b \,, \\ \delta\tilde{\mathscr{H}}^a &= \xi^a\, F + \xi^{ab}\, F_{1b} - \xi \tilde{F}^{1a} \,. \end{aligned} \tag{3.68}$$

The explicit symplectic realisation of the N_4 generators together with the computation of the vector kinetic matrix can be found in appendix.

3.5 $T_3 \times T_3$ IIA orientifold with $D6$-branes

Solvable algebra of global symmetries. The embedding of the $\mathrm{sl}(2,\mathbb{R}) + \mathrm{so}(6,6)$ algebra inside $e_{7(7)}$ is defined by the following identification of the simple roots

$$\begin{aligned} \beta_1 &= \epsilon_1 - \epsilon_2 \,, \\ \beta_2 &= \epsilon_2 - \epsilon_3 \,, \\ \beta_3 &= \epsilon_3 + \epsilon_4 \,, \\ \beta_4 &= -\epsilon_4 + \epsilon_5 \,, \\ \beta_5 &= -\epsilon_5 + \epsilon_6 \,, \\ \beta_6 &= a + \epsilon_4 \end{aligned} \tag{3.69}$$

for the $\mathrm{so}(6,6)$ factor, and

$$\beta = a + \epsilon_1 + \epsilon_2 + \epsilon_3 \,, \tag{3.70}$$

for the $\mathrm{sl}(2,\mathbb{R})$ one. The correspondence axion-root is quite simple and is summarised in table 15.

The triple grading refers to three $\mathrm{O}(1,1)$ groups generated by H_β, H_{λ^3}, H_{λ^6} and parametrised by the moduli $\beta \cdot h$, h_3, h_6:

$$\begin{aligned} \mathrm{O}(1,1)_0 \to e^{\beta\cdot h} &= V_3\, e^{-\phi/4} \,, \\ \mathrm{O}(1,1)_1 \to e^{h_3} &= (V_3)^{1/3}\, (V_3')^{1/3}\, e^{\phi/2} \,, \\ \mathrm{O}(1,1)_2 \to e^{h_6} &= (V_3')^{1/3}\, e^{-3/4\,\phi} \,. \end{aligned} \tag{3.71}$$

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The generators T^{ia}, T_{ab}, T^i_a and T^{ij} form now a 24-dimensional solvable subalgebra N_6 of Solv(so(6, 6)):

$$N_6 = B_{ia}\, T^{ia} + C^{ab}\, T_{ab} + C^a_i\, T^i_a + C_{ij}\, T^{ij}\,, \tag{3.72}$$

whose algebraic structure is encoded in the non-vanishing commutators

$$\begin{aligned} \left[T_{ab},\, T^{ic}\right] &= T^i_{[a}\delta^c_{b]}\,, \\ \left[T^{ia},\, T^j_b\right] &= T^{ij}\,\delta^a_b\,. \end{aligned} \tag{3.73}$$

A possible choice for the coset representative is then

$$L = e^{C_{ij}\,T^{ij}}\, e^{C^a_i\,T^i_a}\, e^{B_{ia}\,T^{ia}}\, e^{C^{ab}\,T_{ab}}\, e^{c\,T}\,\mathbb{E}, \tag{3.74}$$

with $\mathbb{E}$ parameterising the submanifold

$$\mathrm{O}(1,1)_0 \times \frac{\mathrm{GL}(3,\mathbb{R})}{\mathrm{SO}(3)} \times \frac{\mathrm{GL}(3,\mathbb{R})}{\mathrm{SO}(3)}\,. \tag{3.75}$$

Under an infinitesimal transformation $\xi_{ij}\,T^{ij} + \xi^a_i\,T^i_a + \xi_{ia}\,T^{ia} + \xi^{ab}\,T_{ab}$ of N_6 the variation of the axionic scalars is

$$\begin{aligned} \delta C^a_i &= \xi^a_i + \xi^{ab}\,B_{ib}\,, \\ \delta C_{ij} &= \xi_{ij} + \xi_{a[i}\,C^a_{j]}\,, \\ \delta B_{ia} &= \xi_{ia}\,, \\ \delta C^{ab} &= \xi^{ab}\,. \end{aligned} \tag{3.76}$$

Vector fields. The vector fields of this model are G^i_μ,$C^i_\mu = \epsilon^{ijk}\,C_{jk\mu}$, $B_{a\mu}$, $C^a_\mu = \epsilon^{abc}\,C_{bc\mu}$, and we name the corresponding field strengths $\mathscr{F}^i_{\mu\nu}$, $F^i_{\mu\nu}$, $\mathscr{H}_{a\mu\nu}$, $F^a_{\mu\nu}$. The symplectic section of the field strengths and their duals is

$$\{\mathscr{F}^i_{\mu\nu},\, F^i_{\mu\nu},\, \mathscr{H}_{a\mu\nu},\, F^a_{\mu\nu},\, \tilde{\mathscr{F}}_{i\mu\nu},\, \tilde{F}_{i\mu\nu},\, \tilde{\mathscr{H}}^a_{\mu\nu},\, \tilde{F}_{a\mu\nu}\}\,, \tag{3.77}$$

and in table 16 we give their $\mathrm{O}(1,1)^3$ gradings and the corresponding $E_{7(7)}$ weights.

The action of an infinitesimal duality transformation $\xi_{ij}\,T^{ij} + \xi^a_i\,T^i_a + \xi_{ia}\,T^{ia} + \xi^{ab}\,T_{ab} + \xi\,T$ on the symplectic section is

$$\begin{aligned} \delta\mathscr{F}^i &= 0\,, \\ \delta F^i &= \xi\,\mathscr{F}^i\,, \\ \delta\mathscr{H}_a &= \xi_{ia}\,\mathscr{F}^i\,, \\ \delta F^a &= \xi_{ab}\,\mathscr{H}_b + \xi^a_i\,\mathscr{F}^i\,, \\ \delta\tilde{\mathscr{F}}_i &= -\xi_{ia}\,\tilde{\mathscr{H}}^a - \xi^a_i\,\tilde{F}_a - 2\,\xi_{ij}\,F^j - \xi\,\tilde{F}_i\,, \\ \delta\tilde{F}_i &= \xi_{ia}\,F^a + \xi^a_i\,\mathscr{H}_a + 2\,\xi_{ij}\,\mathscr{F}^j\,, \\ \delta\tilde{\mathscr{H}}^a &= \xi^{ab}\,\tilde{F}_a + \xi^a_i\,F^i + \xi\,F^a\,, \\ \delta\tilde{F}_a &= \xi_{ia}\,F^i + \xi\,\mathscr{H}_a\,. \end{aligned} \tag{3.78}$$

The explicit symplectic realisation of the N_6 generators together with the computation of the vector kinetic matrix can be found in appendix.

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Sp-section	$\mathrm{O}(1,1)^3$-grading	weight
$\mathscr{F}^i_{\mu\nu}$	$(-1,-1,-1/2)$	$-\epsilon_i - 1/\sqrt{2}\epsilon_7$
$F^i_{\mu\nu}$	$(1,-1,-1/2)$	$w + \epsilon_j + \epsilon_k$
$\mathscr{H}_{a\mu\nu}$	$(-1,0,-1/2)$	$\epsilon_a - 1/\sqrt{2}\epsilon_7$
$F^a_{\mu\nu}$	$(-1,0,1/2)$	$w + \epsilon_b + \epsilon_c$
$\tilde{\mathscr{F}}_{i\mu\nu}$	$(1,1,1/2)$	$\epsilon_i + 1/\sqrt{2}\epsilon_7$
$\tilde{F}^i_{\mu\nu}$	$(-1,1,1/2)$	$-w - \epsilon_j - \epsilon_k$
$\tilde{\mathscr{H}}^a_{\mu\nu}$	$(1,0,1/2)$	$-\epsilon_a + 1/\sqrt{2}\epsilon_7$
$\tilde{F}_{a\mu\nu}$	$(1,0,-1/2)$	$-w - \epsilon_b - \epsilon_c$

Table 16: Field strengths, $\mathrm{O}(1,1)^3$ gradings, and corresponding weights.

GL(5)-rep.	generator	root	field	dim.
—	$T_{(0,0,0)}$	$\{\epsilon_i - \epsilon_j,\, \epsilon_a - \epsilon_b\}$ $(i<j)$	$\{g_{ij}\}$	10
$\mathbf{5}_{(0,0,1)}$	T^i	$a + \epsilon_i$	C_i	5
$\mathbf{5}_{(0,1,0)}$	T'^i	$\epsilon_i + \epsilon_6$	$B_{i6} \equiv B_i$	5
$\mathbf{10}_{(0,1,1)}$	T^{ij}	$a + \epsilon_i + \epsilon_j + \epsilon_6$	$C_{ij6} \equiv C_{ij}$	10
$\mathbf{1}_{(2,0,0)}$	T	β	$C_{\mu\nu 6} \equiv c$	1

Table 17: Axionic fields for the $T_5 \times T_1$ IIA orientifold, generators of Solv(so(6,6)), $\mathrm{O}(1,1)^3$ gradings, and $\mathrm{GL}(5,\mathbb{R})$ representations.

3.6 $T_5 \times T_1$ IIA orientifold with $D8$-branes

Solvable algebra of global symmetries. The embedding of the $\mathrm{sl}(2,\mathbb{R}) + \mathrm{so}(6,6)$ algebra inside $e_{7(7)}$ is defined by the following identification of the simple roots

$$\begin{aligned} \beta_1 &= \epsilon_1 - \epsilon_2\,, \\ \beta_2 &= \epsilon_2 - \epsilon_3\,, \\ \beta_3 &= \epsilon_3 - \epsilon_4\,, \\ \beta_4 &= \epsilon_4 - \epsilon_5\,, \\ \beta_5 &= \epsilon_5 + \epsilon_6\,, \\ \beta_6 &= a + \epsilon_5\,, \end{aligned} \tag{3.79}$$

for the so(6,6) factor, and

$$\beta = a + \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_5\,, \tag{3.80}$$

for the $\mathrm{sl}(2,\mathbb{R})$ one. The correspondence axion-root is quite simple and is summarised in table 17.

The triple grading refers to three O(1,1) groups generated by H_β, H_{λ^5}, H_{λ^6}, all commuting with $\mathrm{SL}(5,\mathbb{R})$, and parametrised by the moduli $\beta\cdot h$, h_5, h_6:

$$\begin{aligned} \mathrm{O}(1,1)_0 &\to e^{\beta\cdot h} = V_5\, e^{\phi/4}\,, \\ \mathrm{O}(1,1)_1 &\to e^{h_5} = (V_5)^{1/5}\, V_1\, e^{\frac{\phi}{2}}, \\ \mathrm{O}(1,1)_2 &\to e^{h_6} = (V_5)^{1/5}\, e^{-3/4\,\phi}\,. \end{aligned} \tag{3.81}$$

The generators T'^i, T^i and T^{ij} form now a twenty-dimensional solvable subalgebra N_8 of $\mathrm{Solv}(\mathrm{so}(6,6))$:

$$N_8 = B_{i6}\, T'^i + C_i\, T^i + C_{ij}\, T^{ij}\,, \tag{3.82}$$

whose algebraic structure is encoded in the non-vanishing commutator

$$\left[T^i, T'^j\right] = T^{ij}\,. \tag{3.83}$$

A possible choice for the the coset representative is then

$$L = e^{C_{ij}\, T^{ij}}\, e^{B_{i6}\, T'^i}\, e^{C_i\, T^i}\, e^{c\, T}\, \mathbb{E}, \tag{3.84}$$

with the $\mathbb{E}$ parameterising the submanifold:

$$\mathrm{O}(1,1)_0 \times \mathrm{O}(1,1)_1 \times \mathrm{O}(1,1)_2 \times \frac{\mathrm{SL}(5,\mathbb{R})}{\mathrm{SO}(5)}\,. \tag{3.85}$$

Under an infinitesimal transformation $\xi_{ij}\, T^{ij} + \xi_i\, T^i + \xi'_i\, T'^i$ of N_8 the variation of the axionic scalars is

$$\begin{aligned} \delta C_{ij} &= \xi_{[i}\, B_{j]6} + \xi_{ij}\,, \\ \delta B_{i6} &= \xi'_i\,, \\ \delta C_i &= \xi_i\,. \end{aligned} \tag{3.86}$$

Vector fields. The vector fields of this model are G^i_μ, $C_{i6\mu}$, C_μ, $B_{6\mu}$, and we name the corresponding field strengths $\mathscr{F}^i_{\mu\nu}$, $F_{i6\mu\nu}$, $F_{\mu\nu}$, $\mathscr{H}_{6\mu\nu}$. The symplectic section of the field strengths and their duals is

$$\{\mathscr{F}^i_{\mu\nu},\, F_{i6\mu\nu},\, F_{\mu\nu},\, \mathscr{H}_{6\mu\nu},\, \tilde{\mathscr{F}}_{i\mu\nu},\, \tilde{F}^{i6}_{\mu\nu},\, \tilde{F}_{\mu\nu},\, \tilde{\mathscr{H}}^6_{\mu\nu}\}\,, \tag{3.87}$$

and in table 18 we give their $\mathrm{O}(1,1)^3$ gradings and the corresponding $E_{7(7)}$ weights.

The action of an infinitesimal transformation $\xi_{ij}\, T^{ij} + \xi_i\, T^i + \xi'_i\, T'^i + \xi\, T$ on the symplectic section is

$$\begin{aligned} \delta\mathscr{F}^i &= 0\,, \\ \delta F_{i6} &= \xi_i\, \mathscr{H}_6 - \xi'_i\, F + \xi_{ij}\, \mathscr{F}^j\,, \\ \delta F &= -\xi_i\, \mathscr{F}^i\,, \\ \delta\mathscr{H}_6 &= \xi'_i\, \mathscr{F}^i\,, \\ \delta\tilde{\mathscr{F}}_i &= \xi_i \tilde{F} - \xi'_i \tilde{\mathscr{H}}^6 + \xi_{ij} \tilde{F}^{j6} + \xi F_{i6}\,, \\ \delta\tilde{F}^{i6} &= \xi\, \mathscr{F}^i\,, \\ \delta\tilde{F} &= \xi'_i\, \tilde{F}^{i6} + \xi\, \mathscr{H}_6\,, \\ \delta\tilde{\mathscr{H}}^6 &= -\xi_i\, \tilde{F}^{i6} + \xi\, F\,. \end{aligned} \tag{3.88}$$

Sp-section	$\mathrm{O}(1,1)^3$-grading	weight
$\mathscr{F}^i_{\mu\nu}$	$(-1,-1,-1/2)$	$-\epsilon_i - 1/\sqrt{2}\epsilon_7$
$F_{i6\mu\nu}$	$(-1,1,1/2)$	$w + \epsilon_i + \epsilon_6$
$F_{\mu\nu}$	$(-1,-1,1/2)$	w
$\mathscr{H}_{6\mu\nu}$	$(-1,1,-1/2)$	$\epsilon_6 - 1/\sqrt{2}\epsilon_7$
$\tilde{\mathscr{F}}_{i\mu\nu}$	$(1,1,1/2)$	$\epsilon_i + 1/\sqrt{2}\epsilon_7$
$\tilde{F}^{i6}_{\mu\nu}$	$(1,-1,-1/2)$	$-w - \epsilon_i - \epsilon_6$
$\tilde{F}_{\mu\nu}$	$(1,1,-1/2)$	$-w$
$\tilde{\mathscr{H}}^6_{\mu\nu}$	$(1,-1,1/2)$	$-\epsilon_6 + 1/\sqrt{2}\epsilon_7$

Table 18: Field strengths, $\mathrm{O}(1,1)^3$ gradings, and corresponding weights.

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The explicit symplectic realisation of the N_8 generators, together with the computation of the vector kinetic matrix can be found in appendix.

4. Fluxes and gauged supergravity: local Peccei-Quinn symmetry as gauged duality transformations

In the present section we consider the deformation of $\mathcal{N} = 4$ supergravity induced by the presence of fluxes. We shall restrict our analysis, here, only to IIB orientifolds with some (three-form) fluxes turned on, while we shall defer the study of more general fluxes and of the gauge structure of other models elsewhere.

Differently to what happened in the well-studied $T_6/\mathbb{Z}_2$ orientifolds, non-abelian gauged supergravities (for the bulk sector) now emerge, due to the presence of gauge fields originating from the ten-dimensional metric, and of axionic scalars associated to the NS-NS two-form B.

4.1 The $T_4 \times T_2$ IIB orientifold model

In this model, the allowed three-form fluxes are $H^{\lambda}_{ij} = \{H_{aij},\, F_{aij}\}$, and are in correspondence with the representation $(\mathbf{4},\mathbf{6})_{+2}$ of $\mathrm{SO}(2,2) \times \mathrm{GL}(4,\mathbb{R})$. The grading simply counts the number of indices along the internal T_4 and, more specifically, is associated to the subgroup $\mathrm{O}(1,1)_1 \subset \mathrm{GL}(4,\mathbb{R})$. As mentioned in the introduction, inspection of the dimensionally reduced three-form kinetic term indicates for the four-dimensional theory a gauge group $\mathscr{G}_g$ with connection $\Omega_g = X_i\, G^i_\mu + X_\lambda A^\lambda_\mu$ and the following structure:

$$[X_i,\, X_j] = H^{\lambda}_{ij} X_\lambda\,. \tag{4.1}$$

We may identify the gauge generators with isometries as follows:

$$\begin{aligned} X_i &= -H^{\lambda}_{ij}\, T^{j}_{\lambda}\,, \\ X^\lambda &= \frac{1}{2}\, H^{\lambda'}_{ij}\, T^{ij}\,. \end{aligned} \tag{4.2}$$

Using relations (3.19) and the property

$$H^{\lambda}_{k[j} H_{i]\ell\lambda} = \frac{1}{2}\, H^{\lambda}_{ij} H_{k\ell\lambda} - \frac{1}{4}\, H\, \epsilon_{ijkl}\,, \tag{4.3}$$

where $H = H^{\lambda}_{ij}\, H^{ij}_{\lambda}$, one can show that the generators defined in (4.2) fulfil the following algebraic relations

$$[X_i,\, X_j] = H^{\lambda}_{ij}\, X_\lambda - \frac{1}{4} H\, T_{ij}\,, \tag{4.4}$$

which coincide with (4.1) only if $H = 0$ which amounts to the condition that $\int_{T_6} F_{(3)} \wedge H_{(3)} = 0$ (this condition is consistent with a constraint found in [48] on the embedding matrix of a new gauge group in the $\mathcal{N} = 8$ theory, which seems to yield an $\mathcal{N} = 8$ "lifting" of the type IIB orientifold models $T_{p-3} \times T_{9-p}$ discussed here). Under this condition the gauge group is indeed contained in the isometry group of the scalar manifold. Moreover it can be verified that under the duality action of the gauge generators defined in (4.2) the

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vector fields transform in the co-adjoint of the gauge group $\mathscr{G}_g$ and thus provide a consistent definition for the gauge connection Ω_g . The variation of the gauge potentials under an infinitesimal transformation with parameters ξ^λ, ξ^i reads

$$\begin{aligned} \delta A^\lambda_\mu &= \xi^i\, H^\lambda_{ij}\, G^j_\mu + \partial_\mu \xi^\lambda\,, \\ \delta G^i_\mu &= \partial_\mu \xi^i\,, \\ \delta C_{ijk\mu} &= 0\,, \end{aligned} \tag{4.5}$$

and is compatible with the following non-abelian field strengths

$$\begin{aligned} F^\lambda_{\mu\nu} &= \partial_\mu A^\lambda_\nu - \partial_\nu A^\lambda_\mu - H^\lambda_{ij}\, G^i_\mu\, G^j_\nu\,, \\ \mathscr{F}^i_{\mu\nu} &= \partial_\mu G^i_\nu - \partial_\nu G^i_\mu\,, \\ F^i_{\mu\nu} &= \epsilon^{ijkl}\, \left(\partial_\mu C_{jkl\nu} - \partial_\nu C_{jkl\mu}\right)\,. \end{aligned} \tag{4.6}$$

The C_{ij} and Φ^λ_i scalars are also charged and, up to rotations, subject to shifts

$$\begin{aligned} \delta C_{ij} &= \frac{1}{2}\, H^\lambda_{ij}\, \xi_\lambda - \frac{1}{2}\, \xi^k\, H^\lambda_{k[i}\, \Phi_{j]\lambda}, \\ \delta \Phi^\lambda_i &= H^\lambda_{ij}\, \xi^j\,, \end{aligned} \tag{4.7}$$

and their kinetic terms are modified accordingly by covariantisations

$$\begin{aligned} D_\mu C_{ij} &= \partial_\mu C_{ij} - \frac{1}{2}\, H_{ij\,\lambda}\, A^\lambda_\mu + \frac{1}{2}\, G^k_\mu\, H^\lambda_{k[i}\, \Phi_{j]\lambda}, \\ D_\mu \Phi^\lambda_i &= \partial_\mu \Phi^\lambda_i - H^\lambda_{ij}\, G^j_\mu\,. \end{aligned} \tag{4.8}$$

Chern-Simons terms. The gauge group consists of Peccei-Quinn transformations that shift the real part of the vector kinetic matrix $\mathscr{N}$ (the generalised theta angle). In [49, 50], it was shown that such a local transformation is a symmetry of the lagrangian provided suitable generalised Chern-Simons terms are introduced.

In the case at hand, the new contribution to the lagrangian is

$$\mathscr{L}_{\text{c.s.}} \propto \epsilon^{\mu\nu\rho\sigma} \left(H_{\lambda\, ij'}\, A^\lambda_\mu\, G^i_\nu\, \partial_\rho C^{j'}_\sigma + \frac{1}{8}\, H_{\lambda\, ij'}\, H^\lambda_{k\ell}\, G^i_\mu\, C^{j'}_\nu\, G^k_\rho\, G^\ell_\sigma \right)\,, \tag{4.9}$$

corresponding to the non-vanishing entries

$$C_{\lambda,\, ij'} = -H_{\lambda\, ij'} \qquad \text{and} \qquad C_{i,\, \lambda j'} \;=\; H_{\lambda\, ij'}\,, \tag{4.10}$$

where, in general, the coefficients $C_{\Gamma,\Lambda\Sigma}$ define the moduli-independent gauge variation of the real part of the kinetic matrix $\mathscr{N}$

$$\delta_\xi\, \mathrm{Re}\, \mathscr{N}_{\Lambda\Sigma} = \xi^\Gamma\, C_{\Gamma,\Lambda\Sigma}\,. \tag{4.11}$$

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4.2 Type $T_2 \times T_4$ IIB orientifold model

Let us consider the $T_2 \times T_4$ model in presence of the fluxes $H_{ija} = \epsilon_{ij}\, H_a$ and F_{iab}. These fluxes appear as structure constants

$$[X_i,\, X_j] = \epsilon_{ij}\, H_a\, X^a\,,$$
$$[X_i,\, X^a] = F_i^{ab}\, X_b\,, \tag{4.12}$$

of the gauge algebra $\mathscr{G}_g \equiv \{X_i,\, X^a,\, X_a\}$ with connection $\Omega^g_\mu = G^i_\mu\, X_i + B_{a\mu}\, X^a + C^a_\mu\, X_a$, all other commutators vanishing.

The identification

$$X'_i = -F_i^{ab}\, T_{ab} + H_a\, T_i^a\,,$$
$$X^{a\prime} = F_i^{ab}\, T^i_b\,,\;\; X'_a = -H_a\, T\,, \tag{4.13}$$

of the gauge generators with the isometries of the solvable algebra N_5, reproduces only a contracted version of the algebra (4.12) in which three of the central charges X_a vanish and we are left with $X'_a = -H_a\, T$. If we denote by $\{X^\perp\} = \{X_a\}/\{X'_a\}$ these three central generators, we see that the subgroup $\mathscr{G}'_g = \{X'_i,\, X^{a\prime},\, X'_a\}$ of the isometry group which is gauged coincides with the quotient:

$$\mathscr{G}'_g \equiv \mathscr{G}_g/\{X^\perp\}\,, \tag{4.14}$$

that amounts to imposing the vanishing of the central terms on all fields.

On the other hand, transformations generated by the operators in (4.13) induce isometry transformations with parameters:

$$\xi_{ia} = -\xi_i\, H_a\,,$$
$$\xi^{ab} = -\xi^i\, F_i^{ab}\,,$$
$$\xi^a_i = \xi_b\, F_i^{ba}\,,$$
$$\xi = -H_a \xi^a\,, \tag{4.15}$$

where $\xi_i = \epsilon_{ij}\, \xi^j$. Using eqs. (3.49) and (4.15), one can then verify that the vectors G^i_μ, $B_{a\mu}$ and C^a_μ transform in the co-adjoint representation of $\mathscr{G}_g$ under the duality action generated by $\{X_i,\, X^a,\, X_a\}$, so that the above definition of the gauge connection Ω^g_μ is consistent:

$$\delta B_{a\mu} = \xi^i\, G^j_\mu\, \epsilon_{ij}\, H_a + \partial_\mu \xi_a = -\xi_i\, G^i_\mu\, H_a + \partial_\mu \xi_a\,,$$
$$\delta C^a_\mu = \xi^i\, B_{b\mu}\, F_i^{ba} - G^i_\mu\, \xi_b\, F_i^{ba} + \partial_\mu \xi^a\,,$$
$$\delta G^i_\mu = \partial_\mu \xi^i\,. \tag{4.16}$$

Notice that the action of the central charges X_a amounts just to a gauge transformation on C^a_μ. These ten vectors can therefore be used to gauge the group $\mathscr{G}_g$, and the non-abelian field strengths read

$$\mathscr{H}_{a\mu\nu} = \partial_\mu B_{a\nu} - \partial_\nu B_{a\mu} - \epsilon_{ij}\, H_a\, G^i_\mu\, G^j_\nu\,,$$
$$F^a_{\mu\nu} = \partial_\mu C^a_\nu - \partial_\nu C^a_\mu + F_i^{ab}\, G^i_\mu\, B_{b\nu} - F_i^{ab}\, G^i_\nu\, B_{b\mu}\,,$$
$$\mathscr{F}^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu\,. \tag{4.17}$$

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Since $\mathscr{G}_g$ is not part of the global symmetries of the lagrangian, we should restrict ourselves to the quotient G_g, i.e. we demand that central charges $\{T, X_a\}$ vanish on all physical fields. The gauge transformations of the scalar fields

$$\begin{aligned}\delta c &= -H_a\,\xi^a + \xi_a\,F_i^{ab}\,B_{jb}\,\epsilon^{ij}\,,\\ \delta C_i^a &= \xi_b\,F_i^{ba} + \xi^j\,F_j^{ab}\,B_{bi}\,,\\ \delta B_{ia} &= -\xi_i\,H_a\,,\\ \delta C_{ab} &= -\xi^i\,F_{iab}\,,\end{aligned} \tag{4.18}$$

are then compatible with the covariant derivatives

$$\begin{aligned}D_\mu c &= \partial_\mu c + H_a\,C_\mu^a - B_{a\mu}\,F_i^{ab}\,B_{jb}\,\epsilon^{ij}\,,\\ D_\mu C_i^a &= \partial_\mu C_i^a - B_{b\mu}\,F_i^{ba} - G_\mu^j\,F_j^{ab}\,B_{bi}\,,\\ D_\mu B_{ia} &= \partial_\mu B_{ia} + G_{i\mu}\,H_a\,,\\ D_\mu C_{ab} &= \partial_\mu C_{ab} + G_\mu^i\,F_{iab}\,.\end{aligned} \tag{4.19}$$

Chern-Simons terms. Also in this case local Peccei-Quinn transformations demand the inclusion in the lagrangian of the Chern-Simons terms

$$\begin{aligned}\mathscr{L}_{\text{c.s.}} = \epsilon^{\mu\nu\rho\sigma}\Big(&H_a\,G_\mu^i\,C_{i\nu}\,\partial_\rho C_\sigma^a - H_a\,C_\mu^a\,C_{i\nu}\,\partial_\rho G_\sigma^i - \epsilon^{ij}\,F_j^{ab}\,B_{a\mu}\,C_{i\nu}\,\partial_\rho B_{b\sigma}\\ &+\frac{1}{8}\,H_a\,F_k^{ab}\,G_\mu^i\,C_{i\nu}\,G_\rho^k\,B_{b\sigma} - \frac{1}{8}\,\epsilon^{ij}\,H_a\,F_j^{ab}\,B_{b\mu}\,C_{i\nu}\,G_\rho^k\,G_{k\sigma}\Big)\,,\end{aligned} \tag{4.20}$$

corresponding to the non-vanishing components

$$\begin{aligned}C_{i,}{}^j{}_a &= \delta_i^j\,H_a\,,\\ C_{a,i}{}^j &= -\delta_i^j\,H_a\,,\\ C^{a,ib} &= -\epsilon^{ij}\,F_j^{ab}\,,\end{aligned} \tag{4.21}$$

of the $C_{\Gamma,\Lambda\Sigma}$ coefficients.

5. Conclusions and outlooks

In the present paper, we have investigated the symmetries and the structure of several T_6 orientifolds which, in absence of fluxes, have $\mathscr{N} = 4$ supersymmetries in four dimensions. we have not addressed here the question of vacua with some residual supersymmetry, that will be the subject of future investigations. All these models lead to different low-energy supergravity descriptions. When fluxes are turned on, the deformed lagrangian is described by a gauged $\mathscr{N} = 4$ supergravity and fermionic mass-terms and a scalar potential are developed.

The low-energy lagrangians underlying these orientifolds are different versions of gauged $\mathscr{N} = 4$ supergravity with six bulk vector multiplets and additional Yang-Mills multiplets living on the brane world-volume. The gaugings are based on quotients (with respect

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to some central charges) of nilpotent subalgebras of so$(6,6)$. These nilpotent subalgebras are basically generated by the axion symmetries associated to R-R scalars and to NS-NS scalars originating from the two-form B-field.

Along similar lines, one can also consider new examples of orientifolds with $\mathcal{N}=2,1$ four-dimensional supersymmetries, with and/or without fluxes.

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A. Symplectic realisation of the solvable generators

In this appendix, we give the coset representatives of our models in the symplectic basis of vector fields. This is needed in order to compute the kinetic matrix $\mathcal{N}_{\Lambda\Sigma}$, which is a complex symmetric matrix in the space of vectors in the theory. Its imaginary and real parts describe the terms

$$\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}\,F^{\Lambda}_{\mu\nu}F^{\Sigma\,\mu\nu}+\frac{1}{2}\,\mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma}\,\epsilon^{\mu\nu\rho\sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma}\,. \tag{A.1}$$

Model $T_4\times T_2$. The $Sp(24,\mathbb{R})$ representation of the solvable generators in model 1 in the basis (3.33) is:

$$T=\begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&\mathbb{1}&0&0&0&0\\ \eta&0&0&0&0&0\\ 0&0&0&0&0&-\mathbb{1}\\ 0&0&0&0&0&0 \end{pmatrix},$$
$$T=\begin{pmatrix} M^T&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&-M&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix},$$

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$$
T^i_\lambda = \begin{pmatrix} 0 & -(t^i_\lambda)^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(t^i_\lambda \eta)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & t^i_\lambda & 0 & 0 \\ -t^i_\lambda \eta & 0 & 0 & 0 & 0 & 0 \end{pmatrix},
$$

$$
T^{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -t^{ij} & 0 & 0 & 0 \\ 0 & t^{ij} & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.2}
$$

where each block is a 4×4 matrix, $\mathbb{1}$ denotes the identity matrix, $\eta \equiv \eta_{\lambda\lambda'}$ and

$$
(t^i_\lambda)_j{}^{\lambda'} = \delta^i_j \, \delta^{\lambda'}_\lambda \,, \qquad (t^{ij})_{kl} = \delta^i_k \delta^j_l - \delta^i_l \delta^j_k \,. \tag{A.3}
$$

The coset representative is

$$
L = \exp(C_{ij} T^{ij}) \, \exp(\Phi^\lambda_i T^i_\lambda) \, \exp(cT) \, \mathbb{E} = \begin{pmatrix} \mathsf{A} & 0 \\ \mathsf{C} & \mathsf{D} \end{pmatrix}, \tag{A.4}
$$

where $\mathbb{E}$ parametrises the manifold

$$
\mathbb{E} \in \mathrm{O}(1,1)_0 \times \frac{\mathrm{SO}(2,2)}{\mathrm{SO}(2) \times \mathrm{SO}(2)} \times \frac{\mathrm{GL}(4,\mathbb{R})}{\mathrm{SO}(4)}\,, \tag{A.5}
$$

and can be written in the following general form:

$$
\mathbb{E} = \begin{pmatrix} e^{-\varphi} E_{(\ell)} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-\varphi} E & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\varphi} E & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\varphi} \eta E_{(\ell)} \eta & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\varphi} E^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\varphi} E^{-1} \end{pmatrix}, \tag{A.6}
$$

with

$$
\begin{aligned}
E_{(\ell)}{}^\lambda{}_{\hat\sigma} &\in \frac{\mathrm{SO}(2,2)}{\mathrm{SO}(2) \times \mathrm{SO}(2)}\,, \\
E^i{}_{\hat\jmath} &\in \frac{\mathrm{GL}(4,\mathbb{R})}{\mathrm{SO}(4)}\,, \\
e^{\varphi H} &\in \mathrm{O}(1,1)_0\,,
\end{aligned} \tag{A.7}
$$

the hatted indices being the rigid ones transforming under the isotropy group. The blocks is L read

$$
\mathsf{A} = \begin{pmatrix} e^{-\varphi} E_{(\ell)}{}^\lambda{}_{\hat\sigma} & -e^{-\varphi} \Phi^\lambda_i E^i{}_{\hat\jmath} & 0 \\ 0 & e^{-\varphi} E^i{}_{\hat\jmath} & 0 \\ 0 & c\, e^{-\varphi} E^i{}_{\hat\jmath} & e^{\varphi} E^i{}_{\hat\jmath} \end{pmatrix},
$$

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$$
\begin{aligned}
\mathsf{C} &= \begin{pmatrix} c\,e^{-\varphi}\,E_{(\ell)\lambda\hat{\sigma}} & -c\,e^{-\varphi}\,\Phi_{\lambda j}\,E^{j}{}_{\hat{\imath}} & -e^{\varphi}\,\Phi_{\lambda j}\,E^{j}{}_{\hat{\imath}} \\ c\,e^{-\varphi}\,\Phi_{\delta i}\,E_{(\ell)}{}^{\delta}{}_{\hat{\sigma}} & -c\,e^{-\varphi}\,2\,\tilde{C}_{ij}E^{j}{}_{\hat{k}} & -e^{\varphi}\,2\,\tilde{C}_{ij}E^{j}{}_{\hat{k}} \\ -e^{-\varphi}\,\Phi_{\delta i}\,E_{(\ell)}{}^{\delta}{}_{\hat{\sigma}} & e^{-\varphi}\,2\,\tilde{C}_{ij}E^{j}{}_{\hat{k}} & 0 \end{pmatrix}, \\
\mathsf{D} &= \begin{pmatrix} e^{\varphi}\,E_{(\ell)\lambda}{}^{\hat{\sigma}} & 0 & 0 \\ e^{\varphi}\,\Phi^{\lambda}_{i}E_{(\ell)\lambda}{}^{\hat{\sigma}} & e^{\varphi}\,E^{-1}{}_{i}{}^{\hat{\jmath}} & -c\,e^{-\varphi}\,E^{-1}{}_{i}{}^{\hat{\jmath}} \\ 0 & 0 & e^{-\varphi}\,E^{-1}{}_{i}{}^{\hat{\jmath}} \end{pmatrix}, \\
\tilde{C}_{ij} &= C_{ij} + \frac{1}{4}\,\Phi^{\lambda}_{i}\,\Phi_{\lambda j}\,.
\end{aligned} \tag{A.8}
$$

In the sequel we shall need also the expression of A^{-1}:

$$
\mathsf{A}^{-1} = \begin{pmatrix} e^{\varphi}\,E_{(\ell)}{}^{\hat{\sigma}}{}_{\lambda} & e^{\varphi}\,E_{(\ell)}{}^{\hat{\sigma}}{}_{\lambda}\Phi^{\lambda}_{i} & 0 \\ 0 & e^{\varphi}\,E^{-1\hat{\imath}}{}_{j} & 0 \\ 0 & -e^{-\varphi}\,c\,E^{-1\hat{\imath}}{}_{j} & e^{-\varphi}\,E^{-1\hat{\imath}}{}_{j} \end{pmatrix}. \tag{A.9}
$$

In terms of the matrices h, f

$$
f = \frac{1}{\sqrt{2}}\,\mathsf{A}\,, \qquad h = \frac{1}{\sqrt{2}}\,(\mathsf{C} - \mathrm{i}\,\mathsf{D})\,, \tag{A.10}
$$

the kinetic matrix is expressed as (see [51] and references therein)

$$
\mathcal{N} = hf^{-1} = \begin{pmatrix} N_{\lambda\lambda'} & N_{\lambda i} & N'_{\lambda i} \\ * & N_{ij} & N'_{ij} \\ * & * & N''_{ij} \end{pmatrix}, \tag{A.11}
$$

and is characterised by the following entries:

$$
\begin{aligned}
N_{\lambda\lambda'} &= -\mathrm{i}\,e^{2\varphi}\,E_{(\ell)\lambda}{}^{\hat{\sigma}}\,E_{(\ell)\lambda'}{}^{\hat{\sigma}} + c\,\eta_{\lambda\lambda'}\,, \\
N_{\lambda i} &= -\mathrm{i}\,e^{2\varphi}\,E_{(\ell)\lambda}{}^{\hat{\sigma}}\,E_{(\ell)\lambda'}{}^{\hat{\sigma}}\,\Phi^{\lambda'}_{i} + c\,\Phi_{\lambda i} \\
N'_{\lambda i} &= -\Phi_{\lambda i}\,, \\
N_{ij} &= -\mathrm{i}\left((e^{2\varphi} + e^{-2\varphi}\,c^{2})\,E^{-1}{}_{i}{}^{\hat{\imath}}E^{-1}{}_{j\hat{\imath}} + e^{2\varphi}\,\Phi^{\lambda}_{i}\,E_{(\ell)\lambda}{}^{\hat{\sigma}}\,E_{(\ell)\lambda'}{}^{\hat{\sigma}}\,\Phi^{\lambda'}_{j}\right) + c\,\Phi^{\lambda}_{i}\,\Phi_{\lambda j}\,, \\
N'_{ij} &= \mathrm{i}\,c\;e^{-2\varphi}E^{-1}{}_{i}{}^{\hat{\imath}}E^{-1}{}_{j\hat{\imath}} - 2\,\tilde{C}_{ij}\,, \\
N''_{ij} &= -\mathrm{i}\,e^{-2\varphi}\,E^{-1}{}_{i}{}^{\hat{\imath}}E^{-1}{}_{j\hat{\imath}}\,.
\end{aligned} \tag{A.12}
$$

Model $T_2 \times T_4$. The $Sp(24, \mathbb{R})$ representation of the solvable generators in model 2 in the basis (3.48) is:

$$
T' = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \epsilon_{ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \epsilon_{ij} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 \end{pmatrix},
$$

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$$T_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta^{cd}_{ab} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta^{cd}_{ab} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T^{ia} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta^a_b\,\delta^i_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\delta^i_j\,\delta^a_b & 0 \\ 0 & 0 & 0 & -\epsilon^{ij}\,\delta^a_b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\delta^a_b\,\epsilon^{ij} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T^i_a = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\delta^b_a\,\delta^i_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta^i_j\,\delta^b_a \\ 0 & 0 & \epsilon^{ij}\,\delta^b_a & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta^b_a\,\epsilon^{ij} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.13}$$

The coset representative has the form:

$$L = e^{c'\,T'}\,e^{c\,T}\,e^{B_{ia}\,T^{ia}}\,e^{C^a_i\,T^i_a}\,e^{C_{ab}\,T^{ab}}\,\mathbb{E} = \begin{pmatrix} \mathsf{A} & 0 \\ \mathsf{C} & \mathsf{D} \end{pmatrix}, \tag{A.14}$$

where this time the matrix $\mathbb{E}$ describes the submanifold:

$$\mathbb{E} = \mathrm{O}(1,1)_0 \times \frac{\mathrm{GL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{GL}(4,\mathbb{R})}{\mathrm{SO}(4)}, \tag{A.15}$$

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and has the following form:

$$
\begin{aligned}
\mathbb{E} &= \begin{pmatrix}
e^{-\varphi}\,E_2{}^i{}_{\hat{j}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & e^{\varphi}\,E_2^{-1}{}_i{}^{\hat{j}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & e^{-\varphi}\,E_4^{-1}{}_a{}^{\hat{b}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & e^{-\varphi}\,E_4{}^a{}_{\hat{b}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & e^{\varphi}\,E_2^{-1}{}_i{}^{\hat{j}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & e^{-\varphi}\,E_2{}^i{}_{\hat{j}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & e^{\varphi}\,E_4{}^a{}_{\hat{b}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\varphi}\,E_4^{-1}{}_a{}^{\hat{b}}
\end{pmatrix}, \\
E_2{}^i{}_{\hat{j}} &\in \frac{\mathrm{GL}(2,\mathbb{R})}{\mathrm{SO}(2)}\,, \\
E_4{}^a{}_{\hat{b}} &\in \frac{\mathrm{GL}(4,\mathbb{R})}{\mathrm{SO}(4)}\,, \\
e^{\varphi\,H} &\in \mathrm{O}(1,1)_0\,.
\end{aligned} \tag{A.16}
$$

The blocks A, C, D of L can be conveniently described in terms of the following matrices $(\mathbf{B})_{ia} = B_{ia}$, $(\mathbf{C})_i{}^a = C^a_i$, $(\mathscr{C})^{ab} = C^{ab}$:

$$
\begin{aligned}
\mathsf{A} &= \begin{pmatrix}
e^{-\varphi}\,E_2 & 0 & 0 & 0 \\
e^{-\varphi}\,c'\,\epsilon\,E_2 & e^{\varphi}\,E_2^{-1} & 0 & 0 \\
e^{-\varphi}\,\mathbf{B}^t\,E_2 & 0 & e^{-\varphi}\,E_4^{-1} & 0 \\
-e^{-\varphi}\,\mathbf{C}^t\,E_2 & 0 & e^{-\varphi}\,\mathscr{C}\,E_4^{-1} & e^{-\varphi}\,E_4
\end{pmatrix}, \\
\mathsf{C} &= \begin{pmatrix}
e^{-\varphi}c'(c\epsilon+\mathbf{B}\mathbf{C}^t)E_2 & -e^{\varphi}(c\epsilon+\mathbf{B}\mathbf{C}^t)\epsilon E_2^{-1} & e^{-\varphi}c'(\mathbf{C}-\mathbf{B}\mathscr{C})E_4^{-1} & -e^{-\varphi}c'\mathbf{B}E_4 \\
-e^{-\varphi}\epsilon(c\epsilon+\mathbf{B}\mathbf{C}^t)E_2 & 0 & -e^{-\varphi}\epsilon(\mathbf{C}-\mathbf{B}\mathscr{C})E_4^{-1} & e^{-\varphi}\epsilon\mathbf{B}E_4 \\
-e^{-\varphi}c'\mathbf{C}^tE_2 & e^{\varphi}\mathbf{C}^t\epsilon E_2^{-1} & e^{-\varphi}c'\mathscr{C}E_4^{-1} & e^{-\varphi}c'E_4 \\
e^{-\varphi}c'\mathbf{B}^tE_2 & -e^{\varphi}\mathbf{B}^t\epsilon E_2^{-1} & e^{-\varphi}c'E_4^{-1} & 0
\end{pmatrix}, \\
\mathsf{D} &= \begin{pmatrix}
e^{\varphi}\,E_2^{-1} & e^{-\varphi}\,c'\,\epsilon\,E_2 & -e^{\varphi}\,\mathbf{B}\,E_4 & e^{\varphi}\,(\mathbf{C}-\mathbf{B}\,\mathscr{C})\,E_4^{-1} \\
0 & e^{-\varphi}\,E_2 & 0 & 0 \\
0 & 0 & e^{\varphi}\,E_4 & e^{\varphi}\,\mathscr{C}\,E_4^{-1} \\
0 & 0 & 0 & e^{\varphi}\,E_4^{-1}
\end{pmatrix},
\end{aligned} \tag{A.17}
$$

it is also useful to compute A^{-1}:

$$
\mathsf{A}^{-1} = \begin{pmatrix}
e^{\varphi}\,E_2^{-1} & 0 & 0 & 0 \\
-e^{-\varphi}\,c'\,E_2\,\epsilon & e^{-\varphi}\,E_2 & 0 & 0 \\
-e^{\varphi}\,E_4\,\mathbf{B}^t & 0 & e^{\varphi}\,E_4 & 0 \\
e^{\varphi}\,E_4^{-1}\,(\mathscr{C}\,\mathbf{B}^t+\mathbf{C}^t) & 0 & -e^{\varphi}\,E_4^{-1}\,\mathscr{C} & e^{\varphi}\,E_4^{-1}
\end{pmatrix}. \tag{A.18}
$$

We then compute the kinetic matrix $\mathscr{N}$ whose independent components are:

$$
\mathscr{N} = \begin{pmatrix}
N_{ij} & N_i{}^j & N_i{}^a & N_{ia} \\
* & N^{ij} & N^{ia} & N^i{}_a \\
* & * & N^{ab} & N^a{}_b \\
* & * & * & N_{ab}
\end{pmatrix}, \tag{A.19}
$$

where

$$
\begin{aligned}
N_{ij} = -\mathrm{i}\Big[& E_2^{-1}{}_i{}^{\hat{\jmath}}\,E_2^{-1}{}_j{}^{\hat{\jmath}}\,(e^{2\,\varphi}+e^{-2\,\varphi}\,c'^2) + e^{2\,\varphi}\,B_{ia}\,E_4{}^a{}_{\hat{a}}\,E_4{}^b{}_{\hat{a}}\,B_{jb} + \\
& + e^{2\,\varphi}\,(-B_{ic}\,C^{ca}+C^a_i)\,E_4^{-1}{}_a{}^{\hat{a}}\,E_4^{-1}{}_b{}^{\hat{a}}\,\left(C^{bd}\,B_{jd}+C^b_j\right)\Big] - 2\,B_{a(i}\,C^a_{j)}\,c'\,,
\end{aligned}
$$

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$$
\begin{aligned}
N_i{}^j &= -\mathrm{i}\, e^{-2\varphi}\, c'\, \epsilon_{ik}\, E_2{}^k{}_{\hat{k}}\, E_2{}^j{}_{\hat{k}} + c\, \delta_i{}^j - B_{ia}\, C^a_k\, \epsilon^{kj}\,, \\
N_i{}^a &= \mathrm{i}\, e^{2\varphi} \left[B_{ib}\, E_4{}^b{}_{\hat{b}}\, E_4{}^a{}_{\hat{b}} + (-B_{ib}\, C^{bc} + C^c_i)\, E_4^{-1}{}_c{}^{\hat{c}}\, E_4^{-1}{}_d{}^{\hat{c}}\, C^{da} \right] + c'\, C^a_i\,, \\
N_{ia} &= -\mathrm{i}\, e^{2\varphi}\, (-B_{ib}\, C^{bc} + C^c_i)\, E_4^{-1}{}_c{}^{\hat{c}}\, E_4^{-1}{}_a{}^{\hat{c}} - c'\, B_{ia}\,, \\
N^{ij} &= -\mathrm{i}\, e^{-2\varphi}\, E_2{}^i{}_{\hat{k}}\, E_2{}^j{}_{\hat{k}}\,, \\
N^{ia} &= -\epsilon^{ij}\, C^a_j\,, \\
N^i{}_a &= \epsilon^{ij}\, B_{ja}\,, \\
N^{ab} &= -\mathrm{i}\, e^{2\varphi} \left(-C^{ad}\, E_4^{-1}{}_d{}^{\hat{d}}\, E_4^{-1}{}_c{}^{\hat{d}}\, C^{cb} + E_4{}^a{}_{\hat{b}}\, E_4{}^b{}_{\hat{b}} \right)\,, \\
N^a{}_b &= -\mathrm{i}\, e^{2\varphi}\, C^{ad}\, E_4^{-1}{}_d{}^{\hat{d}}\, E_4^{-1}{}_b{}^{\hat{d}} + c'\, \delta^a{}_b\,, \\
N_{ab} &= -\mathrm{i}\, e^{2\varphi}\, E_4^{-1}{}_a{}^{\hat{d}}\, E_4^{-1}{}_b{}^{\hat{d}}\,.
\end{aligned}
\tag{A.20}
$$

Model $T_1 \times T_5$. The $Sp(24, \mathbb{R})$ representation of the N_4 generators is the following:

$$
T^a = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta^a_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \delta^a_b & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\delta^a_b & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta^a_b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
T_a = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \delta^b_a & 0 & 0 & 0 & 0 \\
0 & 0 & -\delta^b_a & 0 & 0 & 0 & 0 & 0 \\
0 & -\delta^b_a & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta^b_a & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
T_{ab} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\delta^{cd}_{ab} & 0 & 0 & 0 & 0 \\
0 & 0 & \delta^{cd}_{ab} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

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$$
T = \begin{pmatrix} 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mathbb{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mathbb{1} & 0 \end{pmatrix} . \tag{A.21}
$$

We have chosen the coset representative to have the form given in eq. (3.64). We may choose for the matrix $\mathbb{E}$ the following matrix form:

$$
\mathbb{E} = \begin{pmatrix} e^{\varphi}\, E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-\varphi}\, E & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\varphi}\, E^{-1}{}_a{}^{\hat b} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\varphi}\, E^{-1}{}_a{}^{\hat b} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\varphi}/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\varphi}/E & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{-\varphi}\, E^a{}_{\hat b} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\varphi}\, E^a{}_{\hat b} \end{pmatrix} , \tag{A.22}
$$

where:

$$
\begin{aligned}
& E^a{}_{\hat b}\,, \qquad E \in \mathrm{O}(1,1)_1 \times \mathrm{O}(1,1)_2 \times \frac{\mathrm{SL}(5,\mathbb{R})}{\mathrm{SO}(5)}\,, \\
& e^{H\,\varphi} \in \mathrm{O}(1,1)_0\,.
\end{aligned} \tag{A.23}
$$

The blocks $\{\mathsf{A},\, \mathsf{C},\, \mathsf{D}\}$ of L and A^{-1} have the following form:

$$
\begin{aligned}
\mathsf{A} &= \begin{pmatrix} E\, e^{\varphi} & c\, E\, e^{-\varphi} & 0 & 0 \\ 0 & E\, e^{-\varphi} & 0 & 0 \\ B_a\, E\, e^{\varphi} & c\, B_a\, E\, e^{-\varphi} & e^{\varphi}\, E^{-1}{}_a{}^{\hat b} & c\, e^{-\varphi}\, E^{-1}{}_a{}^{\hat b} \\ 0 & B_a\, e^{-\varphi} & 0 & e^{-\varphi}\, E^{-1}{}_a{}^{\hat b} \end{pmatrix} , \\
\mathsf{C} &= \begin{pmatrix} 0 & B_a\, C^a\, E\, e^{-\varphi} & 0 & e^{-\varphi}\, (B_b\, C^{ba} + C^a)\, E^{-1}{}_a{}^{\hat d} \\ -B_a\, C^a\, E\, e^{\varphi} & -c\, B_a\, C^a\, E\, e^{-\varphi} & -e^{\varphi}\, (B_b\, C^{ba} + C^a)\, E^{-1}{}_a{}^{\hat d} & -e^{-\varphi}\, c\, (B_b\, C^{ba} + C^a)\, E^{-1}{}_a{}^{\hat d} \\ 0 & -e^{-\varphi}\, C^a\, E & 0 & e^{-\varphi}\, C^{ab}\, E^{-1}{}_b{}^{\hat d} \\ C^a\, E\, e^{\varphi} & c\, C^a\, E\, e^{-\varphi} & e^{\varphi}\, C^{ab}\, E^{-1}{}_b{}^{\hat d} & e^{-\varphi}\, c\, C^{ab}\, E^{-1}{}_b{}^{\hat d} \end{pmatrix} , \\
\mathsf{D} &= \begin{pmatrix} e^{-\varphi}/E & 0 & -B_a\, e^{-\varphi}\, E^a{}_{\hat d} & 0 \\ -c\, e^{-\varphi}/E & e^{\varphi}/E & c\, B_a\, e^{-\varphi}\, E^a{}_{\hat d} & -B_a\, e^{\varphi}\, E^a{}_{\hat d} \\ 0 & 0 & e^{-\varphi}\, E^a{}_{\hat d} & 0 \\ 0 & 0 & -c\, e^{-\varphi}\, E^a{}_{\hat d} & e^{\varphi}\, E^a{}_{\hat d} \end{pmatrix} , \\
\mathsf{A}^{-1} &= \begin{pmatrix} e^{-\varphi}/E & -c\, e^{-\varphi}/E & 0 & 0 \\ 0 & e^{\varphi}/E & 0 & 0 \\ -e^{-\varphi}\, E_{\hat d}{}^a\, B_a & c\, e^{-\varphi}\, E_{\hat d}{}^a\, B_a & e^{-\varphi}\, E_{\hat d}{}^a & -c\, e^{-\varphi}\, E_{\hat d}{}^a \\ 0 & -e^{\varphi}\, E_{\hat d}{}^a\, B_a & 0 & e^{\varphi}\, E_{\hat d}{}^a \end{pmatrix} ,
\end{aligned} \tag{A.24}
$$

from equations (A.10) and (A.11) we compute the matrix $\mathscr{N}$:

$$
\mathscr{N} = \begin{pmatrix} N & N_1 & N^{(1a)} & N^a \\ * & N_{1\,1} & N_1{}^{(1a)} & N_1{}^a \\ * & * & N^{(1a)\,(1b)} & N^{(1a)\,b} \\ * & * & * & N^{a\,b} \end{pmatrix} , \tag{A.25}
$$

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whose entries are

$$
\begin{aligned}
N &= -\mathrm{i}\, e^{-2\,\varphi}\, \left(B_a\, B_b\, E^a{}_{\hat a}\, E^b{}_{\hat a} + \frac{1}{E^2}\right) , \\
N_1 &= \mathrm{i}\, c\, e^{-2\,\varphi}\, \left(B_a\, B_b\, E^a{}_{\hat a}\, E^b{}_{\hat a} + \frac{1}{E^2}\right) , \\
N^{(1a)} &= \mathrm{i}\, e^{-2\,\varphi}\, B_b\, E^a{}_{\hat a}\, E^b{}_{\hat a} \,, \\
N^a &= C^a + B_b\, C^{ba} - \mathrm{i}\, c\, e^{-2\,\varphi}\, B_b\, E^a{}_{\hat a}\, E^b{}_{\hat a} \,, \\
N_{1\,1} &= -\mathrm{i}\, \left(e^{-2\,\varphi}\, c^2 + e^{2\,\varphi}\right)\, \left(B_a\, B_b\, E^a{}_{\hat a}\, E^b{}_{\hat a} + \frac{1}{E^2}\right) , \\
N_1{}^{(1a)} &= -\left(B_b C^{ba} + C^a\right) - \mathrm{i}\, c\, e^{-2\,\varphi}\, B_b\, E^a{}_{\hat a}\, E^b{}_{\hat a} \,, \\
N_1{}^a &= \mathrm{i}\, \left(e^{-2\,\varphi}\, c^2 + e^{2\,\varphi}\right)\, B_b\, E^a{}_{\hat a}\, E^b{}_{\hat a} \,, \\
N^{(1a)\,(1b)} &= -\mathrm{i}\, e^{-2\,\varphi}\, E^a{}_{\hat a}\, E^b{}_{\hat a} \,, \\
N^{(1a)\,b} &= -C^{ab} + \mathrm{i}\, e^{-2\,\varphi}\, c\, E^a{}_{\hat a}\, E^b{}_{\hat a} \,, \\
N^{a\,b} &= -\mathrm{i}\, \left(e^{-2\,\varphi}\, c^2 + e^{2\,\varphi}\right)\, E^a{}_{\hat a}\, E^b{}_{\hat a} \,,
\end{aligned}
\tag{A.26}
$$

where the asterisks denote the symmetric entries.

Model $T_3 \times T_3$. The $Sp(24,\mathbb{R})$ representation of the N_6 generators is the following:

$$
T_{ab} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \delta^{cd}_{ab} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta^{cd}_{ab} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} ,
$$

$$
T^{ia} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta^a_b \delta^i_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\delta^i_j \delta^a_b & 0 \\
0 & 0 & 0 & \delta^i_j \delta^a_b & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \delta^a_b \delta^i_j & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} ,
$$

$$
T^i_a = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta^b_a \delta^i_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta^i_j \delta^b_a \\
0 & 0 & \delta^i_j \delta^b_a & 0 & 0 & 0 & 0 & 0 \\
0 & \delta^b_a \delta^i_j & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} ,
$$

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$$
T^{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\,\delta^{ij}_{kl} & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\,\delta^{ij}_{kl} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},
$$

$$
T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathbb{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.27}
$$

We have chosen the coset representative to have the form given in eq. (3.74). We may choose for the matrix $\mathbb{E}$ the following matrix form:

$$
\mathbb{E} = \begin{pmatrix} e^{-\varphi}\,E^{i}_{1\,\hat{\jmath}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\varphi}\,E^{i}_{1\,\hat{\jmath}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-\varphi}\,E_2^{-1}{}_a{}^{\hat{b}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\varphi}\,E^{a}_{2\,\hat{b}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\varphi}\,E_1^{-1}{}_i{}^{\hat{\jmath}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\varphi}\,E_1^{-1}{}_i{}^{\hat{\jmath}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{\varphi}\,E^{a}_{2\,\hat{b}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\varphi}\,E_2^{-1}{}_a{}^{\hat{b}} \end{pmatrix},
$$

where

$$
\begin{aligned}
E^{i}_{1\,\hat{\jmath}} &\in \left(\frac{\mathrm{GL}(3,\mathbb{R})}{\mathrm{SO}(3)}\right)_1, \\
E^{a}_{2\,\hat{b}} &\in \left(\frac{\mathrm{GL}(3,\mathbb{R})}{\mathrm{SO}(3)}\right)_2, \\
e^{\varphi\,H} &\in \mathrm{SO}(1,1)_0\,.
\end{aligned} \tag{A.28}
$$

The blocks $\{\mathsf{A},\,\mathsf{C},\,\mathsf{D}\}$ of L and A^{-1} have the following form:

$$
\mathsf{A} = \begin{pmatrix} e^{-\varphi}\,E^{i}_{1\,\hat{\jmath}} & 0 & 0 & 0 \\ c\,e^{-\varphi}\,E^{i}_{1\,\hat{\jmath}} & e^{\varphi}\,E^{i}_{1\,\hat{\jmath}} & 0 & 0 \\ B_{ia}\,e^{-\varphi}\,E^{i}_{1\,\hat{\jmath}} & 0 & e^{-\varphi}\,E_2^{-1}{}_a{}^{\hat{b}} & 0 \\ C^{a}_{i}\,e^{-\varphi}\,E^{i}_{1\,\hat{\jmath}} & 0 & e^{-\varphi}\,C^{ab}\,E_2^{-1}{}_b{}^{\hat{c}} & e^{-\varphi}\,E_2{}^{a}{}_{\hat{c}} \end{pmatrix},
$$

$$
\mathsf{C} = \begin{pmatrix} -2\,c\,e^{-\varphi}\,\tilde{C}_{ij}\,E^{j}_{1\,\hat{k}} & -2\,e^{\varphi}\,\tilde{C}_{ij}\,E^{j}_{1\,\hat{k}} & -c\,e^{-\varphi}\,(C^{a}_{i} + B_{ib}\,C^{ba})\,E_2^{-1}{}_a{}^{\hat{c}} & -e^{-\varphi}\,c\,B_{ib}\,E_2^{-1\,b}{}_{\hat{d}} \\ 2e^{-\varphi}\,\tilde{C}_{ij}\,E^{j}_{1\,\hat{k}} & 0 & e^{-\varphi}\,(C^{a}_{i} + B_{ib}\,C^{ba})\,E_2^{-1}{}_a{}^{\hat{c}} & e^{-\varphi}\,B_{ia}\,E_2{}^{a}{}_{\hat{c}} \\ c\,e^{-\varphi}\,C^{a}_{i}\,E^{i}_{1\,\hat{k}} & e^{\varphi}\,C^{a}_{i}\,E^{i}_{1\,\hat{k}} & c\,e^{-\varphi}\,C^{ab}\,E_2^{-1}{}_b{}^{\hat{c}} & c\,e^{-\varphi}\,E_2{}^{a}{}_{\hat{c}} \\ c\,e^{-\varphi}\,B_{ia}\,E^{i}_{1\,\hat{k}} & e^{\varphi}\,B_{ia}\,E^{i}_{1\,\hat{k}} & c\,e^{-\varphi}\,E_2^{-1}{}_a{}^{\hat{c}} & 0 \end{pmatrix},
$$

$$
\mathsf{D} = \begin{pmatrix} e^{\varphi}\,E_1^{-1}{}_i{}^{\hat{\jmath}} & -c\,e^{-\varphi}\,E_1^{-1}{}_i{}^{\hat{\jmath}} & -e^{\varphi}\,B_{ia}\,E^{a}_{2\,\hat{d}} & -e^{\varphi}\,(C^{a}_{i} + B_{ib}\,C^{ba})\,E_2^{-1}{}_a{}^{\hat{c}} \\ 0 & e^{-\varphi}\,E_1^{-1}{}_i{}^{\hat{\jmath}} & 0 & 0 \\ 0 & 0 & e^{\varphi}\,E^{a}_{2\,\hat{d}} & e^{\varphi}\,C^{ab}\,E_2^{-1}{}_b{}^{\hat{c}} \\ 0 & 0 & 0 & e^{\varphi}\,E_2^{-1}{}_a{}^{\hat{c}} \end{pmatrix},
$$

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$$
\begin{aligned}
\mathsf{A}^{-1} &= \begin{pmatrix} e^{\varphi}\, E_1^{-1}{}^{\hat{\jmath}}{}_i & 0 & 0 & 0 \\ -c\, e^{-\varphi}\, E_1^{-1}{}^{\hat{\jmath}}{}_i & e^{-\varphi}\, E_1^{-1}{}^{\hat{\jmath}}{}_i & 0 & 0 \\ -e^{\varphi}\, E_{2\hat{a}}{}^{a}\, B_{ia} & 0 & e^{\varphi}\, E_{2\hat{a}}{}^{a} & 0 \\ -e^{\varphi}\, E_2^{-1}{}^{\hat{c}}{}_a\, (B_{ib}\, C^{ba} + C_i^a) & 0 & -e^{\varphi}\, E_2^{-1}{}^{\hat{c}}{}_a\, C^{ab} & e^{\varphi}\, E_2^{-1}{}^{\hat{c}}{}_a \end{pmatrix}, \\
\tilde{C}_{ij} &= C_{ij} + \frac{1}{2}\, C_i^a\, B_{ja}\,.
\end{aligned} \tag{A.29}
$$

The vector kinetic matrix can now be calculated and has the following form:

$$
\mathscr{N} = \begin{pmatrix} N_{ij} & N'_{ij} & N_i{}^a & N_{ia} \\ * & N''_{ij} & N'^{a}_i & N'_{ia} \\ * & * & N^{ab} & N^a{}_b \\ * & * & * & N_{ab} \end{pmatrix}, \tag{A.30}
$$

whose entries are

$$
\begin{aligned}
N_{ij} &= 2\, c\, B_{a(i}\, C^a_{j)} - \mathrm{i} \Big[(c^2\, e^{-2\varphi} + e^{2\varphi})\, E_1^{-1}{}_i{}^{\hat{\jmath}}\, E_1^{-1}{}_j{}^{\hat{\jmath}} + e^{2\varphi}\, (C_i^a + B_{ib}\, C^{ba}) \times \\
&\qquad \times (C_j^c + B_{jb}\, C^{bc})\, E_2^{-1}{}_a{}^{\hat{c}}\, E_2^{-1}{}_c{}^{\hat{c}} + e^{2\varphi}\, B_{ia}\, B_{jb}\, E^a_{2\,\hat{c}}\, E^b_{2\hat{c}} \Big], \\
N'_{ij} &= -2\, \tilde{C}_{ij} + \mathrm{i}\, c\, e^{-2\varphi}\, E_1^{-1}{}_i{}^{\hat{\jmath}}\, E_1^{-1}{}_j{}^{\hat{\jmath}}\,, \\
N_i{}^a &= -c\, C_i^a + \mathrm{i}\, e^{2\varphi} \left[(C_i^b + B_{ic}\, C^{cb})\, C^{ad}\, E_2^{-1}{}_b{}^{\hat{e}}\, E_2^{-1}{}_d{}^{\hat{e}} + B_{ib}\, E^a_{2\,\hat{c}}\, E^b_{2\hat{c}} \right], \\
N_{ia} &= -c\, B_{ia} + \mathrm{i}\, e^{2\varphi}\, (C_i^b + B_{ic}\, C^{cb})\, E_2^{-1}{}_b{}^{\hat{e}}\, E_2^{-1}{}_a{}^{\hat{e}}\,, \\
N''_{ij} &= -\mathrm{i}\, e^{-2\varphi}\, E_1^{-1}{}_i{}^{\hat{\jmath}}\, E_1^{-1}{}_j{}^{\hat{\jmath}} \\
N'^{a}_i &= C_i^a\,, \\
N'_{ia} &= B_{ia}\,, \\
N^{ab} &= -\mathrm{i}\, e^{2\varphi} \left[E^a_{2\,\hat{c}}\, E^b_{2\hat{c}} + C^{ac}\, C^{bd}\, E_2^{-1}{}_c{}^{\hat{e}}\, E_2^{-1}{}_d{}^{\hat{e}} \right], \\
N^a{}_b &= c - \mathrm{i}\, e^{2\varphi}\, C^{ac}\, E_2^{-1}{}_c{}^{\hat{e}}\, E_2^{-1}{}_b{}^{\hat{e}}\,, \\
N_{ab} &= -\mathrm{i}\, e^{2\varphi}\, E_2^{-1}{}_a{}^{\hat{e}}\, E_2^{-1}{}_b{}^{\hat{e}}\,.
\end{aligned} \tag{A.31}
$$

Model $T_5 \times S_1$. The $\mathrm{Sp}(24, \mathbb{R})$ representation of the N_8 generators is the following:

$$
T^i = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta^i_j & 0 & 0 & 0 & 0 \\ -\delta^i_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta^i_j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\delta^i_j & 0 & 0 \end{pmatrix},
$$

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$$
T'^{i} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\delta^i_j & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta^i_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta^i_j \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \delta^i_j & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
T^{ij} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta^{ij}_{kl} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \delta^{ij}_{kl} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
T = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}. \tag{A.32}
$$

We have chosen the coset representative to have the form given in eq. (3.84). We may choose for the matrix $\mathbb{E}$ the following matrix form

$$
\mathbb{E} = \begin{pmatrix}
e^{-\varphi}\, E^i{}_{\hat{\jmath}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & e^{-\varphi}\, E^{-1}{}_i{}^{\hat{\jmath}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & e^{-\varphi}\, E & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & e^{-\varphi}/E & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & e^{\varphi}\, E^{-1}{}_i{}^{\hat{\jmath}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & e^{\varphi}\, E^i{}_{\hat{\jmath}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & e^{\varphi}/E & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\varphi}\, E
\end{pmatrix}, \tag{A.33}
$$

where:

$$
\begin{aligned}
E^i{}_{\hat{\jmath}},\, E &\in \mathrm{O}(1,1)_1 \times \mathrm{O}(1,1)_2 \times \frac{\mathrm{SL}(5,\mathbb{R})}{\mathrm{SO}(5)}\,, \\
e^{\varphi\, H} &\in \mathrm{SO}(1,1)_0\,.
\end{aligned} \tag{A.34}
$$

The blocks $\{\mathsf{A},\, \mathsf{C},\, \mathsf{D}\}$ of L and A^{-1} have the following form

$$
\mathsf{A} = e^{-\varphi} \begin{pmatrix}
E^i{}_{\hat{\jmath}} & 0 & 0 & 0 \\
(B_i\, C_j + C_{ij})\, E^j{}_{\hat{\jmath}} & E^{-1}{}_i{}^{\hat{\jmath}} & -B_i\, E & C_i/E \\
-C_i\, E^i{}_{\hat{\jmath}} & 0 & E & 0 \\
B_i\, E^i{}_{\hat{\jmath}} & 0 & 0 & 1/E
\end{pmatrix},
$$

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$$
\begin{aligned}
\mathsf{C} &= e^{-\varphi}\begin{pmatrix} c\,(B_i\,C_j + C_{ij})\,E^j{}_{\hat{\jmath}} & c\,E^{-1}{}_i{}^{\hat{\jmath}} & -c\,B_i\,E & c\,C_i/E \\ c\,E^i{}_{\hat{\jmath}} & 0 & 0 & 0 \\ c\,B_i\,E^i{}_{\hat{\jmath}} & 0 & 0 & c/E \\ -c\,C_i\,E^i{}_{\hat{k}} & 0 & c\,E & 0 \end{pmatrix}, \\
\mathsf{D} &= e^{\varphi}\begin{pmatrix} E^{-1}{}_i{}^{\hat{\jmath}} & (B_i\,C_j + C_{ij})\,E^j{}_{\hat{\jmath}} & C_i/E & -B_i\,E \\ 0 & E^i{}_{\hat{\jmath}} & 0 & 0 \\ 0 & B_i E^i{}_{\hat{\jmath}} & 1/E & 0 \\ 0 & -C_i E^i{}_{\hat{\jmath}} & 0 & E \end{pmatrix}, \\
\mathsf{A}^{-1} &= e^{\varphi}\begin{pmatrix} E^{-1\hat{\jmath}}{}_i & 0 & 0 & 0 \\ E_{\hat{\imath}}{}^j\,(C_j\,B_i - C_{ji}) & E_{\hat{\imath}}{}^j & E_{\hat{\imath}}{}^j\,B_j & -E_{\hat{\imath}}{}^j\,C_j \\ C_i/E & 0 & 1/E & 0 \\ -B_i\,E & 0 & 0 & E \end{pmatrix}.
\end{aligned}
$$

The vector kinetic matrix can now be calculated and has the following form

$$
\mathcal{N} = \begin{pmatrix} N_{ij} & N_i{}^j & N_i & N'_i \\ * & N^{ij} & N^i & N'^i \\ * & * & N & N' \\ * & * & * & N'' \end{pmatrix}, \tag{A.35}
$$

whose entries are

$$
\begin{aligned}
N_{ij} &= -\mathrm{i}\,e^{2\,\varphi}\Big[E^{-1}{}_i{}^{\hat{\jmath}}\,E^{-1}{}_j{}^{\hat{\jmath}} + (B_i\,C_k + C_{ik})\,(B_j\,C_n + C_{jn})\,E^k{}_{\hat{\ell}}\,E^n{}_{\hat{\ell}} + \\
&\qquad + \frac{1}{E^2}\,C_i\,C_j + B_i\,B_j\,E^2\Big], \\
N_i{}^j &= c - \mathrm{i}\,e^{2\,\varphi}\,\Big[(B_i\,C_k + C_{ik})\,E^k{}_{\hat{\ell}}\,E^j{}_{\hat{\ell}}\Big], \\
N_i &= -\mathrm{i}\,e^{2\,\varphi}\,\Big[(B_i\,C_k + C_{ik})\,E^k{}_{\hat{\ell}}\,B_j\,E^j{}_{\hat{\ell}} + \frac{1}{E^2}\,C_i\Big], \\
N'_i &= \mathrm{i}\,e^{2\,\varphi}\,\Big[(B_i\,C_k + C_{ik})\,E^k{}_{\hat{\ell}}\,C_j\,E^j{}_{\hat{\ell}} + E^2\,B_i\Big], \\
N^{ij} &= -\mathrm{i}\,e^{2\,\varphi}\,E^i{}_{\hat{\ell}}\,E^j{}_{\hat{\ell}}, \\
N^i &= -\mathrm{i}\,e^{2\,\varphi}\,B_j\,E^i{}_{\hat{\ell}}\,E^j{}_{\hat{\ell}}, \\
N'^i &= \mathrm{i}\,e^{2\,\varphi}\,C_j\,E^i{}_{\hat{\ell}}\,E^j{}_{\hat{\ell}}, \\
N &= -\mathrm{i}\,e^{2\,\varphi}\,\Big[B_i\,B_j\,E^i{}_{\hat{\ell}}\,E^j{}_{\hat{\ell}} + \frac{1}{E^2}\Big], \\
N' &= c + \mathrm{i}\,e^{2\,\varphi}\,C_i\,B_j\,E^i{}_{\hat{\ell}}\,E^j{}_{\hat{\ell}}, \\
N'' &= -\mathrm{i}\,e^{2\,\varphi}\,\big[C_i\,C_j\,E^i{}_{\hat{\ell}}\,E^j{}_{\hat{\ell}} + E^2\big]\,.
\end{aligned} \tag{A.36}
$$

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